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ABSTRACT

This study investigated problem solving skills in mechanics problems that required the use of diagrams. These skills were examined in two ways. First, the study examined student problem solving skills using solution scripts from the Western Australian Tertiary Admission Examination in physics. Solution attempts by students in the 1978 and 1979 physics papers were used as an information base, to identify and classify student difficulties in selected mechanics problems. Second, student problem solving skills were further examined to detail the error types in performance on diagram analysis. A population of students attempted to solve selected problems. Their solution attempts followed four seperate methods. Each method of solution was analyzed separately. The findings of this analysis identified many points of difficulty. A further study examined in detail the errors involved in student performance on diagram analysis. Analysis of test results provided information on the initial steps and difficulties for student problem solvers using diagrams as a solution aid. The general findings of this study suggest that student inability to use diagram analysis is a major influence on the lack of mechanics problem solving competence among tertiary admission examinees. There is in this finding an implied demand for a clearer problem solving strategy, where the use of diagram analysis is appropriate. (Author/CW)

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PHYSICS PROBLEM SOLVING:

STUDENT PERFORMANCE ANALYSIS ON MECHANICS PROBLEMS REQUIRING DIAGRAMMATIC VISUALISATION

WILFRED FRANCIS PRENDERGAST

This thesis is presented as part of the requirements of the degree of Master of Applied Science of the Western Australian Institute of Technology.

1984

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ABSTRACT

This study investigated problem solving skills in mechanics problems that required the use of diagrams. These skills were examined in two ways. Firstly, the study examined student problem solving skills using solution scripts from the Western Australian Tertiary Admission Examination in physics. Solution attempts by students in the 1978 and 1979 physics papers were used as an information base, to identify and classify student difficulties in selected mechanics problems. Secondly, student problem solving skills were further examined to detail the error types in performance on diagram analysis.

The purpose of using Tertiary Admission physics scripts was to identify difficulties encountered by students in attempting to solve a complex non-routine mechanics problem under examination conditions.

A sample of 700 student solution scripts was made available for research, from a population of 2536 physics candidates in the 1978 Western Australian Tertiary Admission Examination. Selection of a specific problem - drawbridge problem - for analysis was based on the general level of difficulty that students had experienced. No diagram was provided in the problem statement. This was one of the features which helped identify early general difficulties in mechanics problem solving.

From the 1979 physics paper, a similar mechanics problem was selected. The jibcrane problem was solved under slightly different conditions: namely, provision of a physical diagram and an

opportunity for students to follow more than one basic solution process. In the 1979 Western Australian Tertiary Admission Examination, there were 2604 physics candidates. A population of 2017 students attempted to solve the selected problem. Their solution attempts followed four separate methods. Each method of solution was analysed separately.

Analysis of student solution attempts was based on comparison with a checklist of model solution steps. Each model solution recorded every step of physics-based logic deemed useful in clarifying the problem statement and linking it with its final answer(s).

A step scoring process was developed, within which each step of a student solution attempt was scaled according to its proximity to a correct statement of fact, formula, figures or diagrammatic representation. Separate scores were recorded for total omission, skipped step, incorrect statement and correct statement.

The findings of this analysis identified many points of difficulty:

i) Students found difficulty extracting, summarising, positioning, visualising and comprehending given details and values.

ii) Another early observation of initial solution difficulty was the inability to generate a correct physical diagram by accurate translation from the information of the problem.

iii) Transforming the physical diagram to a freebody diagram of vectors often produced further obstacles to students' solution progress.

iv) Students could not recognise, separate and extract vector values, components and principles from the problem statement.

v) Consistent errors occurred with application of triangle properties, crossmultiplication and simplification of mathematical values.

vi) Students could not put together a resultant vector from a set of analysed components.

vii) Drawing together a two stage formula application became an obstacle to solution progress. This suggested that seeing ahead, beyond the immediate solution step, was in many cases, too far for student thinking.

A further study examined in detail the errors involved in student performance on diagram analysis. This Mechanics Vector Analysis Test was developed and applied in October, 1981. Six weeks before their tertiary admission examination, 168 Year 12 physics students, from four representative schools, completed the test. Analysis of test results provided information on the initial steps and difficulties for student problem solvers using diagrams as a solution aid.

Results obtained from the study as a whole, suggested:

i) that students could not compose a vector resultant from its components;

ii) they could not construct diagrams as a useful aid to mechanics problem solving.

The general findings of this study suggest that student inability to use diagram analysis is a major influence on the lack of mechanics problem solving competence among tertiary admission examinees. There is in this finding an implied demand for a clearer problem solving strategy, where the use of diagram analysis is appropriate.

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CHAPTER 1

GENERAL INTRODUCTION

The search for more effective methods of teaching and learning physics problem solving has taken a long time to attract research attention (Wickelgren, 1974). It is the author's contention that existing methods of problem solving have been presented more in the form of summary advice than as the result of any substantial method analysis or research. The physics text by Giancoli (1974) provides a good example of this point. There is a need for better scientific procedure in the field of problem solving (Reif et al, 1976).

Physicists are problem solvers. Educators (Carin and Sund, 1980), chemists and physicists (Woods, 1978; Lochhead, 1979) have strongly supported this basic belief. The question of how physics students are taught to become successful problem solvers is a more complex issue, about which there is a lesser level of consensus (Lochhead, 1979; Branca, 1980). Indeed, there seem to be more theories and confusions than agreements (Cohen, 1975; Mettes et al, 1981).

The types of problems dealt with by this investigation were specifically selected complex mechanics problems. Their solutions involved mathematical concepts as well as physical understanding. How does one commence a discussion of problem solving?

> "A major difficulty in discussing problem solving seems to be a lack of any clear-cut agreement as to what constitutes a problem."
> (Krulik & Rudnik, 1980, p.3.)

To begin with, the author will define 'problem' in general terms, as a statement of difficulty or perplexity, requiring solution by way of logical explanation. A more specific definition will be introduced later on.

Malone (1980) has concentrated his research in the region of mathematical problem solving. He noted the difficulties associated with the empirical evaluation of student problem solving ability and went into great detail on the difficulties in determining effectiveness of strategies designed to enhance problem solving competence. For a long time, according to a similar project (Wright, 1968), this has been a major stumbling block in developing specialised relevant teaching strategies. Research and development of such a specialised teaching strategy for physics problem solving will be one of the aims of this investigation.

Malone's (1980)findings were that techniques for measurement of student performance evaluation and prediction of student competence are advancing only slowly. The author's own research readings support Malone's conclusions.

The emphasis of this research was to concentrate upon identifying and analysing problem solving competence. Investigation of these areas, it was hoped, would help delineate useful lines of logic in student learning sequences. From this may come recommendations for developmental teaching strategies for physics instruction.

CONCERN OF THE STUDY

This study is specifically concerned with the examination of student ability to solve physics problems at the upper secondary school levels. It uses student tertiary admission examination scripts in physics as a source of information on student physics problem solving competence. This information base has not previously been the subject of any rigorous research evaluation in Western Australia.

The first aim of this study was to identify common major obstacles that physics students encounter in the solution of physics problems under examination conditions, as shown by examiner's reports of Deacon (1980) and Greenhalgh (1976). This information was intended to provide a focus for research into physics problem solving.

The second aim was to classify the multitude of student difficulties into appropriate categories.

The final aim of this study was to select and carry out analytical research on a specific difficulty in student physics problem solving.

BRIEF OUTLINE OF THE STUDY

This investigation used written examination scripts to analyse the physics problem solving performances of Year 12 tertiary admission examination (TAE) candidates. Attention was directed to the methods and errors in solution attempts of two specific TAE mechanics problems.

These problems were judged to be sufficiently complex to require mathematical and symbolic formulation for eventual solution. Commitment to the diagrammatic visualisation of many problem solving components was anticipated.

From the two selected physics problems, detailed analyses of problem solving steps were carried out. This study was designed to determine the degree of influence of different error types on the final outcome of student attempts in solving physics problems.
Initial assessment of error in and omission of solution steps, had the general purpose of identifying and separating short-term from long-term research goals. Some student difficulties were anticipated to show definite promise for short-term investigation results. These regions of difficulty were to provide an immediate research area.

Error and omission analyses classified as having longer-term research goals, could have extended well beyond the confines of this thesis. Such research analyses, which were seen to come within that (longer-term) category, form no part of this thesis.

A decision was made to concentrate the detailed study on problem solving methods involving diagrammatic visualisation of the problem and its solution. There were several reasons for emphasising the visual aspect of physics problem solving. The prime reason was that this seemed to be the region in which examinees encountered their first common major physics obstacle to general progress.

Closely related to the nature of this common obstacle was the visual aspect of student performance. By the presence or absence, clarity or incomplete form, of each diagram, a judgement can be made as to how clear the problem is in the student's mind. The use by students of appropriately distinguishing symbols and labels, may be equally relevant in establishing levels of visual literacy and visual comprehension. It was also a means of establishing how much value each student placed upon visual explanation as part of a logical sequencing. This permitted some evaluation of the strength of visual translation as an integral part of the student physics problem solving approach.

The difficulties that students encounter, in mastering the content of a physics education syllabus, have a direct bearing on competence in problem solving (Clement, 1979). Since physics problem solving depends upon physical and mathematical concepts, comprehension of problem solving processes cannot proceed satisfactorily, until a descriptive analysis and mathematical translation of the underlying principles have been explained, understood and remembered (Wertime, 1979; Kaput, 1979).

The level of formal operations of student problem solvers has previously been researched by Arons (1976) in his experience with a physical science course for preservice and inservice elementary teachers. Arons' initial finding was that no more than 25% of these student teachers had themselves reached the Piagetian formal operational level of thinking (Piaget, 1963).

It was Aron's conviction that students do not attain formal operation intellect through being told about concepts, modes of thought, and lines of reasoning. Rather this level of development seems to arise by way of enquiry-oriented, elementary curricula. These programmes he felt, contained the vital components of formal development. Such components included direct personal experience with phenomena, evidence, inference, concept formation, and quantitative reasoning, as well as experience in verbalising one's own growing insights and perceptions of relationship.

Arons, through his repeated programmes on physical science, identified several procedures and strategies as establishing a framework that impels students toward the new intellectual level of formal operational thought. These procedures and strategies included: exploratory activity and question asking, prior to concept formation and model building.

Translation of words into symbols and symbols into words, especially in converting written problem statements, is recommended by Arons as specifically aiding development of the formal operational intellect. Sketching of physical relations such as velocity-time graphs, assists interpretation of significant problem situations.

Arons (1976) identified specific factors and procedures that seemed to assist attainment of that formal operational level of intellectual development, by students who initially are at the concrete or transitional stage. Finally, the research evidence of Fuller, Karplus and Lawson (1977) suggests that the use of physics and physics problem solving can help develop formal reasoning.

VISUAL TRANSLATION IN PROBLEM SOLVING

Cognitive learning theory (learning with understanding) (Ausubel and Robinson, 1972) views the learner as much more than a passive recorder of external events. The learner is seen as a very active participant within the learning process (Woods et. al, 1977). Selection of information within physics problem solving should be the active role of every learner. Decisions on the importance and relevance of stored memory, whether new information should be stored in general or precise format, determination of storage location, or conducting a search and retrieval process from memory, are some of the vital activities of a participant learner, according to Rubinstein and Pfeiffer (1975).

The old maxim "A picture is worth a thousand words" suddenly becomes a very useful philosophy. Within the framework of summarised and organised mental storage, a clearly labelled sequence of written to visual translations, seems to offer a simple means of long-term storage. The recommendation of Rubinstein and Pfeiffer (1975) is not to rely on memory. They suggested that problem solvers

write	the problem down in its primitive form;
transform	the primitive statement to simpler language;
package	(consolidate) the related information;
translate	the written problem statement to a simpler abstract mathematical statement;
translate	the written problem statement to a simpler, more abstract diagram, chart or graph.

This advice is also supported by Giancoli (1974) in his text book of physics.

All of the above suggestions have specific relevance to this study. Writing, transforming, consolidating and translating understanding, appear to be dependent upon how completely the solution is done and on whether the logic is expressed or implied. The value of technically precise labelling in diagrams and formulae is part of the language of physics problem solving. Language in all its forms is a powerful aid to problem solving (Rubinstein and Pfeiffer, 1975).

The correct question and the appropriate key word may even help the learner to usefully alter a frame of reference. While this is happening, other relevant interrelations may become apparent for a more logical sequence of explanation, or for a better pattern of interacting ideas. In physics problem solving, additionally, the value of technically precise visualisation, with subtle separation of component structures, may permit the right process to formulate itself as a logical consequence.

SUMMARY

This Chapter has provided a general introduction to the details of the study.

The following chapter reviews problem solving literature relevant to the study. Chapter 3 will describe the research design and methodology, giving a rationale for its use. Chapter 4 will present results of research, followed by related discussion on each set of data. Chapter 5 will summarise the research, interpret the results and draw conclusions from the information available.

CHAPTER 2

REVIEW OF BACKGROUND AND RESEARCH LITERATURE

INTRODUCTION

This study seeks to identify, classify and understand the details in student physics problem solving difficulties of tertiary admission examinees in Western Australia. The study will also try to select one or more specific student difficulties for more detailed examination.

This chapter brings attention to literature of a background and research-related nature. Consideration is given to the role of heuristics and terminology useful in the field of physics problem solving.

HISTORICAL REFERENCE IN PROBLEM SOLVING

A body of background references in science and mathematics problem solving has been available for many years and has separated itself into several categories in a variety of academic disciplines. The encouragement of competitive competence in problem solving calculation, has an historical line extending well back into the last century (Polya and Kilpatrick, 1974). Discussions on the definitions for 'problem,' 'problem solving' and 'heuristic,' are considered later in this chapter.

Since the 1940's, under the general guidance of Polya (1944), a general heuristic approach - the ideals of what problem solving should entail - have greatly influenced research in problem solving.

The term heuristic has a more general meaning than algorithm. Algorithm implies an action sequence: a definite technical flowchart or procedure for performing a specified task in problem solving with certainty (Krulik & Rudnick, 1980).

Heuristic guarantees no such certainty in problem solving. It is however, a device, process or rule of thumb (Kilpatrick, 1969) that can improve problem solving performance if used wisely.

As the Polya problem solving heuristic is referred to on several occasions in this thesis, it is presented in detail in Table 1.
Problem solving research has steadily gathered momentum, since the work of Polya and has resulted in broad-ranging attempts to develop common ground approaches to solution methods and strategies (Suchman, 1960; Polya, 1962; Piaget, 1963).From the mid-1970's onward, this great coming together of academic researchers and disciplines, from sociology to physics, has experienced some deceleration.

Despite the efforts of several research groups, 'problem solving' had still persistently refused to release many of its vital secrets involving mental process and intellectual analysis (Gray, 1979; Clement, 1979).

Problem solving appears to have defied all scientific endeavours to obtain a single satisfactory interdisciplinary solution (Cohen, 1975; Simon and Simon, 1979). In some instances, there has been expressed disappointment with the results of well-intentioned, applied experimentation with discovery learning method in the classroom (Cohen, 1975).

HOW TO SOLVE IT

First.
You have to *understand* the problem.

UNDERSTANDING THE PROBLEM

- *What is the unknown? What are the data? What is the condition?*
- Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?
- Draw a figure. Introduce suitable notation.
- Separate the various parts of the condition. Can you write them down?

Second.
Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

DEVISING A PLAN

- Have you seen it before? Or have you seen the same problem in a slightly different form?
- *Do you know a related problem?* Do you know a theorem that could be useful?
- *Look at the unknown!* And try to think of a familiar problem having the same or a similar unknown.
- *Here is a problem related to yours and solved before. Could you use it?* Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?
- Could you restate the problem? Could you restate it still differently? Go back to definitions.
- If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?
- Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

Third.
Carry out your plan.

CARRYING OUT THE PLAN

- Carrying out your plan of the solution *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?

Fourth.
Examine the solution obtained.

LOOKING BACK

- Can you *check the result?* Can you check the argument?
- Can you derive the result differently? Can you see it at a glance?
- Can you use the result, or the method, for some other problem?

TABLE 1: Polya Heuristic (Krulik and Reys, 1980)

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In the 1980's, there is even a hint of reversal of that 'interdisciplinary togetherness' in problem solving research and development. The undercurrent of the trend has recently surfaced in science-oriented publications on teaching and learning. In these papers, dissatisfaction with over-generalised problem solving heuristics is quite clearly expressed in the following publication:

> "Polya developed a well-known set of heuristics for problem solving in mathematics. In science, however, we found that Polya's set of heuristics is unsatisfactory."
> (Mettes, Pilot, Roosink and Kramers-Pals, 1980, p.882.)

Polya's (1954) advice on how to gain problem solving expertise is to solve a large number of them. The conclusions of Polya and others seems to be that his methods just happened to work for them (Greenfield, 1982).

Polya's emphatic advice, through his heuristic for mathematical problem solving, may well be the best available. However, to automatically assume that Polya's heuristic will work for students, is an inappropriate conclusion. Research by Mettes et al. (1980) supports this contention.

We should of course, be just and reasonable in relation to the demands made of Polya's heuristic. In terms of general argument, by definition, no heuristic guarantees problem solving success. Heuristics merely attempt to point out a general guidance system. So Polya's general heuristic has probably been unfairly expected to provide 'doctrines that promise to cure dozens of educational ills out of the same bottle' (Cronbach, 1966).

RESEARCH LITERATURE ON GENERAL PROBLEM SOLVING

That students lack a generalised ability to solve problems has been a world wide observation by educators for a long time, and the problem has become worse as more students enter tertiary education (Dawson, 1978). Student difficulty with problem solving has become more noticeable since the early 1970's (Woods, 1975), when academics at several American universities became sufficiently concerned to do something major about it (Woods, 1974).

That a learner's needs had to be properly identified, seems now to have been a natural point of origin. What may not have been quite so obvious though, is the need for thoughtful, logical sequencing and interpretation of remediation within the general study programme (Woods, Crowe, Wright and Hoffman, 1977; Woods, 1978).

The author would argue that the too-ready acceptability of the Polya heuristic has led to the too-easily dispensed method of teaching by generality and by way of only a few respresentative problem solving examples. This opinion is supported in the work of Whimby (1979), on teaching analytical reasoning in mathematics.

The third statement from the Polya heuristic is most relevant to the considerations of this thesis. "Draw a figure", says Polya. "Introduce suitable notation", he continues. It will be noted that this is the only point where no question is contemplated. It is a command. It is therefore assumed that students can draw a diagram and can introduce suitable notation. Further, there is an implied assumption that physics problem solvers can translate the abstract

concepts of a specific physics problem into a single diagram, appropriate to the needs of further progress. There is implication here that this will lead to effective formulation and eventual solution progress. From this argument alone, it may be seen just how inadequate for specific physics problem solving is such a generalised instruction.

Research and development in the region of physics problem solving is now moving toward emphasis on the skills necessary to process knowledge content (Lin, 1979).

The topic of visual translation seems to be an area of neglect. Currently, more detailed attention has been paid to verbal description, interview techniques, and symbolic formulation. The leap from abstract problem statement directly to abstract solution and explanation is classical, in the author's opinion, for experts explaining to experts. Neither sees any need for an intervening concrete visual clarification. This opinion is supported by the work of Larkin (1981), whose research at the University of Pittsburg clearly differentiates this as a characteristic of the expert problem solving process, as distinct from that of students.

Relevant to this thesis is the finding that novice physics problem solvers commonly omit a diagrammatic translation from the written statement of the problem (Marples, 1974). Student problem solvers frequently attempt to construct equations directly from the written problem statement. Skilled problem solvers are differentiated by their ability to separate the component identification task from the task of relating components quantitatively in equations (Larkin, 1981).

PROBLEM SOLVING AND PHYSICS TEACHING

> "Developing the ability to solve non-routine problems is important at all levels of the formal educational process."
>
> (Malone et al, 1980, p.204.)

Over a long period of educational development, there has been a broad interdisciplinary consensus in support of this statement by many eminent scientists, including Socrates 403 B.C.; Pappus 300 A.D.; Bolzano 1830; Fermi 1936; and Polya, 1944 (Smith, 1958). All spoke extensively in support of developing logic by way of problem solving. More recently, the need for problem solving skills has been emphasised:

> "Problem solving is such an important skill that we must ensure that our students have excellence in this ability."
>
> (Woods, D.R., 1978, p.3.)

It seems appropriate then, and indeed an urgent priority, that the difficulties of teaching and learning problem solving be resolved.

Physics education has traditionally been steeped in problem solving situations related to applied mathematics (Smith, 1958). However, the work of Gerson and Primrose (1977) is strong evidence of the concrete/formal gap that has been allowed to develop between the required formal knowledge of mathematical theory and the concrete to formal transition teaching opportunities that have been bypassed in the teaching of physics.

Well-researched evidence has shown that very few student problem solvers are abstract performers (Dawson, 1978). This is possibly why expert problem solvers are failing in their endeavours to develop successful problem solving process instruction (Cohen, 1975). As far as the student is concerned, the most important explanatory steps in logic are just not present (Woods, 1978).

THE ROLE OF MODELS IN THE PROBLEM SOLVING PROCESS

In current teaching practice, well developed physics curricula need support from soundly evaluated problem solving models, upon which teachers can rely with confidence (Cohen, 1975; Reif et al., 1976). From traditional models, problem solving practitioners can make flexible choice appropriate to a particular audience (Emkey, 1979).

That the Polya or Fermi strategies may not be appropriate starting models for teaching physics problem solving, has been discussed in only a few research publications (Reif et al., 1976; Peters and Miller, 1976; Mettes et al., 1980). The correct choice of models can be important to analysis of learning (Cohen, 1975) and in the commencement for an organised line of attack (Polya, 1956). No model should be regarded as perfect (Polya, 1980).

HEURISTICS AND PHYSICS PROBLEM SOLVING

THE POLYA HEURISTIC

Polya heuristic traditional models for solving problems have suffered from a common characteristic of having been developed by a small number of experts in a specific academic atmosphere, and then put forward as general procedure. The difficulties of teaching solutions to common problems are evident in examiners' reports (Bailey, 1979; Deacon, 1980).

Publications by Polya (1944) have become the traditional commencing reference in problem solving. His text, "How To Solve It", first published in 1944, forms an almost legendary base reference.

Polya has been exemplified as an expert in the field of mathematical problem solving. In this specific area of research, he noted the intrinsic qualities inherent in mathematics and also in the general problem solving process (Polya, 1980).

His sequenced heuristic for developing a solution demanded: understanding of the problem, devising a plan, carrying out the plan, and looking back.

The word heuristic originates with the Greek verb 'Heuriskein - to discover'. The science of discovery, called heuristics, signifies a collection of advisory techniques for discovering solutions to problems. For students, it can become a powerful problem solving mechanism. For the talented and inspired student, this science of discovery can even be classified as an art (Hughes, 1976):

> "When and where heuristic began is unknown. We find it first in geometric analysis as the method Socrates used in order to draw from Meno's unschooled slave a means of duplicating a square. Heuristic maxims...were spelled out by Francois Viete (16th Century) and Rene Descartes (17th Century). In our own time, heuristic has been championed by George Polya."
> (Hughes, 1976, p.v.)

Heuristics tries to look at the ways of thinking through a problem. It analyses method and the logic of that method. The answer, though still a vital end point, is de-emphasised in heuristics, in favour of appreciating those ideas and logical lines of argument that lead a problem solver toward the desired goal. There is no guarantee, however, that the heuristic chosen will solve a particular problem.

As may easily be interpreted from the general sequence of steps, there is nothing that specifically restricts Polya's strategy to mathematics. This is the reason for its popular strength and is, of

course, the base from which his esteem has grown. He is recognised as having developed a problem solving strategy applicable to a very general field of study (Damarin, 1975), even though his basic research emphasised mathematical problem solving.

Polya's publications (Polya, et. al., 1972, 1974) have concentrated a research programme around the results of students who were judged by open competition to be already mathematically superior.

These select students were, it is reasonable to suggest, already operating in the formal mode of problem solving competence (Polya and Kilpatrick, 1974). Polya's mathematical heuristic was thus, more for students already familiar with the tricks and algorithmic techniques needed for a particular problem (Cohen, 1975). His strategy gives instruction in powerful general problem solving terms. The style of presentation is evidenced by the structure of his text, "Mathematics and Plausible Reasoning" (Polya, 1954).

A particular heuristic is first of all explained, and its application to a particular problem is then demonstrated. The student is then confronted with a graded set of problems amenable to solution by way of that method.

It has been claimed by Wickelgren (1974) that such confrontation is where much irrelevant time wasting commences. The cause of student frustration and irritation with problem solving seems to be that there has been no check by the text book or lecturer to find out if the student has been adequately equipped to commence this problem solving confrontation.

Wickelgren strongly supports the idea that the student should have been previously presented with a clearly analysed and systematised set of basic methods in solving calculation problems.

In teaching physics problem solving by example, most teachers and lecturers hope that, in the process of following through individual examples, more generalised skills, methods and strategies of problem solving might be absorbed. It is suggested here then, that Polya did not necessarily propose his heuristic for the general classroom teacher. Rather has he concentrated on the heuristic as a method for solving problems.

The Polya strategy of teaching problem solving in mathematics has research-based evidence to show that it works well for the formal thinking, already competent, mathematical problem solver (Polya and Kilpatrick, 1974).

However, only very recently (Polya, 1980) has attention been directed to its application by teachers in an average classroom (Schoenfeld, 1980) where the majority of students are not necessarily at the formal level of thinking (Dawson, 1978; Lawson, 1979). Such students have not yet acquired the relevant learning skills necessary to adequately handle independently, heuristics of the problem-solving process (Cohen, 1975; Reif et al, 1976; Larkin, 1981).

THE FERMI HEURISTIC

The Fermi heuristic is presented as a contrast with Polya's heuristic. Its presentation is also designed to consider the level of structure appropriate when teaching problem solving techniques.

The acquisition of generalised physics problem solving skills, understanding of unusual problems, analysis of unique difficulties, and synthesis of appropriate solutions, are specific aims of the Fermi Strategy of physics problem solving (Cohen, 1975).

The absence of systematised method is specifically true for Fermi's personalised, alternate discovery and development process of physics problem solving (Cohen, 1975). Very little problem solving direction was provided. No single method is emphasised, and this has an obvious advantage of reducing the rote learned, formula plugging approach to finding a solution.

Used by exponents of discovery learning in physics, the Fermi learning strategy makes use of relatively open-ended questions on problem solving or deals with problem-loaded research topics. A question such as, "How do we walk or run?" provides an idea of the nature of the Fermi problem style.

> "Other Fermi problems have a definite solution but are of a 'non-standard' form requiring the skillful selection and artful utilization of perhaps quite elementary physical models. Teachers wishing to follow the Fermi stratagem face two difficulties. The first is the scarcity of Fermi problems....The second difficulty in introducing Fermi problems is the absence of any comprehensive tutor's guide embodying theoretical analysis and practical experiences in the presentation and effective utilization of such problems. In fact, the casual introduction of Fermi problems into certain innovatory courses of recent years has often lead to obvious failure, as the students participating have lacked any model of how to proceed in tackling any problem other than those more conventional problems which I term 'formula crankers'."
> (Cohen, 1975, pp. 2, 3.)

From Cohen's analysis of the Fermi Strategy, it would appear that, without relevant preservice or inservice training to provide teacher guidance, the Fermi problems become inadequate for purposes of directive guidance in discovery learning.

Polya, it seems, has assumed that there are specific, real and logical pathways that can become generalised problem solving talents, to be developed and utilised with good judgement. Fermi, on the other hand, would persuade us to consider the possibility of alternate realities and a personalised logic in discovery.

This review of the two most widely known problem-solving methods attempts to set one way of solving a problem against the other. It is contended however, that there is nothing to prevent their living alongside one another or indeed, merging.

Both problem solving methods, it is suggested (Cohen, 1975), are suitable for the already talented and competent problem solver of physics or mathematics or any other academic pursuit. In the average classroom, the Polya heuristic, being more group oriented and more directive, seems appropriate to both individualised strategy and to classroom organisation.

The Fermi problem solving strategy seems more suited to tertiary postgraduate programmes. In the average secondary and lower tertiary physics classroom, this method is time-wasting and non-productive (Cohen, 1975), without constant close checking guidance and the availability of small group instruction (Emkey, 1979).

DEVELOPMENTS IN PROBLEM SOLVING RESEARCH

Problem solving skills, techniques and mechanisms have attracted considerable attention from researchers wishing to identify what is needed to become a good problem solver (Lochhead, 1979; Lin, 1979). Research is also being focused upon the intuitive, analytic and synthetic components of creativity brought into play by competent problem solvers (Koplowitz, 1979; Davis, 1980; Woods, 1982). It seems, however, that not a great deal of progress in teaching problem solving strategies, has resulted from all of this research.

Some researchers have focused upon the difficulty of measurement in the problem solving process (Gray, 1979; Malone, 1980). Others have concentrated on student abilities, teaching methods (Whimbey, 1979), problem solving strategies (Schoenfeld, 1979), and student mental readiness for science problem solving in the Piagetian sense (Lochhead, 1979). Finally, there are those researches that have emphasised the effect that individualised learning can have in comparison with classroom presentation (Sparks, 1979; Emkey, 1979).

In the teaching of general learning and problem solving skills, Reif, Larkin and Brackett (1976) concentrated on an attempt to teach general cognitive skills, regarded as important to introductory physics. They noted that, in regard to solution process, many students approached problems in a haphazard and ineffective manner.

For example, many students simply proceeded to calculate something from the information stated in the problem, without ever asking themselves if this calculation will bring them closer to a

reasonable answer. So, even when students are aware that the relevant values and rules necessary for solution were present, there was often an inability to proceed, through lack of a systematic strategy, to guide the application of facts and principles.

Accordingly, Reif, Larkin and Brackett (1976) have tried to teach students a simple general problem solving strategy involving four major steps: description, planning, implementation, and checking. Although this heuristic may appear to be relatively primitive, it did address some of the key difficulties of organisation in student problem solving approach.. In particular, the steps of their systematic approach later remained as essential steps in more complex strategies with more complex problems.

Initially, the strategy was explained to students. Then it was demonstrated with a few illustrating problems. Afterwards, students were given practice with feedback, on a variety of problems encountered during an introductory course.

Based upon detailed observation with small groups of individual students, the preliminary results reported by Reif indicated several effects of the special instruction designed to teach the strategy. Students who received the instruction exhibited in subsequent protocols (written solutions and taped verbal comments) a greater use of diagrams, and also a more intelligent use of algebra before attempting a formula cranking process. There was also evidence of more extensive planning and greater success in attaining solutions by specially instructed students, compared with students taking an ordinary physics course without such instruction.

Finally, Reif noted that, even when specially instructed students did not attain the correct solution to a moderately complex problem, they did tend to apply a better line of reasoning and to record more relevant steps in their attempts. Students without such instruction tended to become lost in 'blind alley' attempts.

Whether it is appropriate to consider providing time for special instruction really depends upon time and manpower available. It is reasonable to state that, in the 'difficult topics' of physics, mathematics and chemistry, there is a common complaint by teachers of insufficient time, provided in the average class programme, to cover material of consequence to any worthwhile depth (Dawson, 1978).

Preliminary research results of the author support Reif et al (1976) in the claim that adequate special instruction time, allocated to physics problem solving, is an effective means of improving problem solving competence. More usefully though, Reif's resultant recommendations provide a pattern for active response by teachers at the classroom level. Important implications from Reif's data noted that many students never acquired adequate problem solving skills from the usual classroom lectures, discussion, texts and problem sets.

TERMINOLOGY OF PROBLEM SOLVING

Generalisation of problem solving was promoted initially by the mathematician Polya, when he revitalised the notion and study of heuristics or problem solving discovery (Polya, 1962). A merging of influence from both mathematical and psychological disciplines, has found common ground in the thinking and learning processes of mathematics education. This has generated a common region of empirical research into problem solving.

There are many difficulties however, with the educational analysis of understanding in relation to problem solving. The difficulty in developing educational problem solving strategies for physics, has multiple origins. One cannot for example, assume that every capable student knows the ramifications and methods of gaining understanding, in either the concrete or the abstract operational sense. Such assumption merely compounds the difficulty of understanding, on the whole subject of achieving a problem solving competence among physics students. Students en masse, just do not possess the relevant learning skills (Reif et al, 1976).

Several ideas and terms related to problem solving have been discussed above in a descriptive manner, in order to introduce the idea that there are basic differences and ambiguities involved with problem solving. In complex physics problem solving, this thesis will argue that specific, sequenced, cognitive process instruction should precede attempts at problem solving by discovery. This it is argued, is a basic difficulty in current instruction strategy.

There is a need for a systematic study in problem solving terminology in order to prevent confusion of meaning. It also suggests a need for research and writing on the principles of problem solving systematics. Such principles may be paralleled with the principles of biosystematics, which have created order out of diversity within the disciplines of zoology and botany.

TERMINOLOGY USED IN THE STUDY

Problem

"What constitutes a problem?" This is, ironically, a source of general uncertainty within the broad ambit of problem solving research.

> "I'm afraid I can't put it more clearly", Alice replied...
> "for I can't understand it myself to begin with."

This quotation from "Alice in Wonderland", expresses a rather concise summary on the state of the art in problem solving instruction (Lochhead and Clement, 1979). It is the author's contention that this lack of understanding arises from researchers' having combined the contributions of too many unrelated disciplines and non-specific ideas within the common consideration of problem solving.

> "A major difficulty in discussing problem solving seems to be a lack of any clear cut agreement as to what constitutes a 'problem'."
> (Krulik and Rudnik, 1980, p.3.)

The above statement implies a rather embarrassing indictment on problem solving research and development. The statement suggests that researchers cannot even describe or define what it is they are solving - that very starting point of problem solving - the problem!

Krulik and Rudnik (1980) have however, committed themselves to a definition for a problem.

> "A problem is a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires solution, and for which the individual sees no apparent or obvious means or path to obtain the solution." (p. 3)

The above definition formulates the type of commitment needed for adequate discussion on the whole philosophy of problem solving. It helps describe what a problem is and how its descriptors interact. Its use of words and phrases provides a platform for debate, arguments for support or disagreement and avenues for reassessment. Such definitions provide springboards from which one can rise to find a better way.

Problem Solving

Definitive statements about problem solving over the last forty years have been used diversely to the point of conflict (Polya, 1956). For purposes of clearer focus on physics problem solving, a narrower projection seemed appropriate. A new definition of problem solving therefore, was developed by the author to provide guidance for this thesis.

Problem solving is a region in the universe of discovery. It can be seen to encompass the invention, research, development and communication of exploratory analytical pathways. The appropriate synthesis of analytical steps links the problem situation through a cumulative sequence of logical statements, around difficulties, avoiding obstacles, and to a desired and reasonable goal or conclusion. This is achieved by way of a clearly communicated and understandable, but not instantaneously obvious, series of explanatory arguments, operations and translations.

In this particular research, the last clause is essential to meet the demands for written and logical evidence of reasoning, which is an integral part of tertiary admission and most other formal examinations or assessments. It also formulates the basis for scoring error occurrence in the marking of solution attempts to the selected problems analysed in this study.

Problem solving is specific in its discovery of method. There seems therefore, to be some logic in anticipating that related and specialised definitions would be appropriate. This might help to clarify the level and type of problem solving under scrutiny in different academic disciplines.

Analysis and Synthesis

In this thesis, physics problem solving analysis is the expression of verbal, visual or symbolic dispersion of the problem situation and its solution into identified separable statements about components of argument, alternate exploratory pathways, relations, interactions or logical connectives, and reasonable or intuitive assumptions.

Synthesis in physics problem solving will be defined as the verbal, visual or symbolic expression of inventive formulations, wherein logical sequences or structures are drawn together, out of the analysed components of the problem situation and its solution steps. This reconstructive summary provides a unique communication or a set of abstract consequential relations, leading through a logical series of interactions, toward a convergent focus or reasonable conclusion, which we call the answer.

A clear understanding about the relevance of analysis and synthesis to physics problem solving, has given greater weight to the significance of this thesis. As has been previously stated, the focus of this research has been directed at diagrammatic visualisation in physics problem solving. In the problems selected and in the follow-up research, visual translation has been seen as essential illustrative evidence of the difficulties students have encountered. The cognitive process of visual analysis and visual synthesis has been, it is now suggested, a vital yet underemphasised region of difficulty, preventing student problem solving progress.

Translation

Bloom's (1970) taxonomy has discussed translation in a general educational sense. The emphases detailed are quite appropriate to physics problem solving, without the need for realignment of any phraseology.

> "TRANSLATION
>
> Comprehension as evidenced by the care and accuracy with which the communication is paraphrased or rendered from one language or form of communication to another. Translation is judged on the basis of faithfulness and accuracy; that is, on the extent to which the material in the original communication has been altered."

A relevant subheading continues

> "[Translation includes] Skill in translating mathematical verbal material into symbolic statements and vice versa."
> (Bloom, Krathwohl and Masia, 1970, p.170.)

The priority emphasis, that translation provides evidence of understanding, is the main thrust of this research.

The term visualise means forming the relevant mental image of a situation, object or abstraction not present to the sight. In

physics problem solving, therefore, diagrammatic visualisation will be defined as: the written expression of an accurately and appropriately labelled diagram or a sequence of component labelled diagrams. This will provide a visual translation from the problem situation, as evidence for understanding and as an integral component explanation for logic or argument, in the cognitive process of solution method.

Solution and Answer

Solution in problem solving is the sequence, pathway, methods or process of argument leading to the required information. In physics problem solving, solution is the cognitive process of analysing a problem situation, followed by the discovery of synthetic pathways that couple appropriate component statements to construct a cumulative translational explanation, leading to a logical conclusion or answer.

Solution is the series of explanatory steps involved in problem solving. Answer is the 'bottom line' in the problem solving explanatory communication. It is the concluding statement in the solution process.

Sequential Diagrammatic Visualisation

This is a part of problem solving sequence and explanation. It may be explained as the visual translation on paper, from one type of diagram to the next.

SUMMARY

The field of contemporary educational research is increasingly concerned with the fundamental issues of student problem solving. It seems reasonable to suggest that clear problem solving terminology is academically intrinsic to the whole study of physics education.

To those involved with teaching strategy and learning achievement, physics problem solving has provided a very definite region of focus (Krulik & Reys, 1980). On closer resolution however, that focal point of physics education may also be seen to contain regions of descriptive confusion. The very recent focus on problem solving competence in relation to those regions of confusion, has given impetus to this study.

Subsequent chapters in this study will explore the actual results of student problem solving difficulties experienced after two years of an upper secondary physics program.

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

INTRODUCTION

The concern of this study as expressed in Chapter 1, is with the level of student ability to solve physics problems in upper secondary school. It is now appropriate to provide a frame of reference for the design of this research. To generate the design of this study, a series of research questions will be presented in the first section of this Chapter. It will be followed by a statement of research hypotheses. A brief summary of methodology is then included, and the research procedure of the study itself is then described in a detailed sequence.

RESEARCH QUESTIONS

Chapters 1 and 2 have revealed several areas of concern in problem solving for upper secondary students of physics. As a consequence, this thesis now raises four questions for consideration. The first and second questions have a general exploratory nature. These two questions require answering before proceeding to questions three and four, which project the prime focus of the Study.

1. What are the specific difficulties that students encounter in solving complex non-routine mechanics problems?

2. Are there difficulties in solving mechanics problems, common to all TAE physics students?

3. To what extent does the proper use of diagrams in the solving of problems facilitate the solution of mechanics problems?

4. Is there a specific procedure for solving mechanics problems using diagrams?

STATEMENT OF RESEARCH HYPOTHESES

To enable the answering of previously stated research questions, some relevant research hypotheses are now presented.

1. That specific difficulties experienced by students in solving complex mechanics problems, may be distinguished, identified and classified.

2. That some difficulties in mechanics problem solving are common to all T A E physics students.

3. That the use of diagrams to help solve mechanics problems can give students a greater chance of processing a successful solution.

SUMMARY OF METHODOLOGY USED IN THE STUDY

A brief summary of the methodology used to examine the above questions is now presented.

1. The research involved analysis of two years of tertiary admission physics examination papers using mechanics problems. The first part of the analysis involved an exploratory research of solution scripts. Its aim was to identify

difficulties encountered by students in attempting to solve a complex non-routine mechanics problem under examination conditions. The second part involved a similar mechanics problem, solved under slightly different conditions; namely, provision of a physical diagram and an opportunity to follow more than one basic solution process. The solution steps to each of the problems used are detailed in later sections of this Chapter.

2. A marking key was developed for the scoring of each of the mechanics problems investigated. Common difficulties were noted from the resulting tabulations and graphical interpretations. Differences in patterns of difficulty were noted for different levels of student ability.

3. Student difficulty with diagrams, as part of the solution process, was noted. The use or absence of diagrams to aid solution attempts was investigated.
   This led to the development of a test instrument designed to study the details of student diagram difficulties.

4. The Study Instrument for Mechanics Vector Analysis was administered in October of 1981. Six weeks before their Tertiary Admission Physics Examination, 168 Year 12 physics students from four representative schools completed the test. Analysis of the test was intended to provide information on the initial steps and difficulties for student problem solvers, using diagrams as a solution aid.

The results obtained permitted conclusions regarding the basic steps in the correct use of diagrams for mechanics problem solving at the upper secondary level of physics education.

POPULATIONS USED

The first part of the study was an exploratory research designed to identify and classify the types of difficulties that students encounter in physics problem solving, under examination conditions at the upper secondary level of education. The study used a sample of student TAE solution scripts in physics as a source of information on student physics problem solving competence. It is believed that this information base has not previously been the subject of any rigorous research evaluation in Western Australia.

The sample consisted of 700 solution scripts from the 1978 Tertiary Admission Examination in physics. As mentioned earlier in this Chapter, the drawbridge mechanics problem had been chosen for detailed analytical investigation on the basis of specific selection criteria. In the sample used, 416 student papers contained attempted solutions to the drawbridge problem, representing 59% of this script sample of the 1978 physics paper.

The second part of the study was a total population script analysis of the jibcrane problem chosen from the 1979 TAE physics paper. This investigation was designed to expand upon the exploratory research of the first part. Some 2,598 solution scripts were examined. In that population, 2,017 students had attempted the jibcrane problem. This represented 78% of student problem solvers in the 1979 Tertiary Admission Examination in physics.

In the third part, or follow-up study, the investigation was designed to specifically identify the detail of error, omission and difficulty in that part of physics problem solving which has been called diagram analysis. The follow-up research was closely related to the TAE level of student performance. To this end, arrangements were made to visit four schools, involving seven classes, each containing about 25 year 12 physic students.

Testing was organised to be held about six weeks prior to their 1981 TAE physics examination. This it was hoped, would permit most students to be in a proper and similar state of readiness for the testing of their ability in diagram analysis. The sample consisted of 168 students from the 1981 final year population in Western Australian metropolitan schools.

PROBLEM SELECTION CRITERIA

The following selection criteria were used to help choose the mechanics problems for the study.

1. In attempting to solve the problem, the student should be committed to recording a logical sequence of explanatory steps to justify an answer to the problem. This requirement enables analysis of student expressions of insight, mathematical competence, diagrammatic translations and communication skills.
2. Of equal importance to selection criteria is the structure and complexity of the problem. The problem statement should present a well defined quantitative problem, from a not too highly focussed situation.

The conceptual difficulty of such a situation should require creative reorganisation of information and rules of physics, to analyse a novel problem statement and to generate a solution sequence.

3. The sequence of ideas in solving the problem should demand visual interpretation of physics, prior to mathematical formulation, substitution and simplification.

4. The problem should contain a sequential interaction built into the process of solution. This should provide a series of related obstacles, which become progressively more difficult to surmount, rather than being merely a sequence of equally difficult hurdles. This would assist in separating different levels of student success in physics problem solving performance.

An important aspect of the selection criteria was found in the physics examiners' reports on the T A[illegible] physics examinations for each of the years 1978 and 1979. The physics problem selected for the first stage of this study is presented in Figure 1.

Selection of the drawbridge problem from the 1978 TAE physics paper was reinforced by comments in the examiner's report (Bailey, 1979).

The report noted the general level of difficulty that students had experienced in solving this mechanics problem.

> "Many students had difficulty...since they could not adopt the "pull out a formula, plug in the numbers" approach, they had to appreciate the physics involved.
> .....

> There was a definite attempt [by the examiner] to graduate the level of difficulty in the question...in section B...up to a more difficult end [i.e. the drawbridge problem]
>
> .....
>
> A number of areas of weakness were also obvious. ... There was a reluctance to draw adequate diagrams. Candidates should note that marks were given for diagrams alone and they should be encouraged to draw diagrams of adequate size and detail.
>
> .....
>
> There was a definite problem with vectors and their resolution...in question 4(b) [drawbridge problem]...units were often omitted."
>
> (Bailey, 1979, p. 2, 3)

One reason then, for selecting the drawbridge problem, was the general level of difficulty that students experienced in solving this problem, as noted in the examiner's report. The second aspect of interest was that the examiner had constructed the drawbridge question to the level of "difficult" problem in order to seek out and test the ability of the approximately top 5% of physics examinees.

The same selection criteria were applied in choosing the second problem. Selection of the jibcrane problem from the 1979 TAE physics paper was designed to further the investigations carried out in the exploratory research. In his report on the Tertiary Admission Examination for 1979, the physics examiner noted that:

> "Questions of an easy, moderate and hard level of difficulty were selected.
>
> Questions of a more demanding nature were used for discrimination among the better students.
>
> [The jibcrane problem] was badly done. Although there were at least three methods of solving this problem, students somehow were unable to understand the compression and tension forces in the jib and tie."
>
> (Deacon, 1980, p. 1, 3)

In general then, the label of "difficult" problem has been based upon the judgement of each examiner, as well as upon the generally poor performance of students in their solution attempts.

THE DRAWBRIDGE PROBLEM AND ITS SOLUTION

The Problem Statement

A castle drawbridge of mass 2×10^3 kg is 60 m long. It is pivoted at one end at the bottom of the castle wall and it can be raised by a chain, which is connected to the drawbridge 40 m from the pivot and to the castle wall 40 m above the pivot. When the tension in the chain is just sufficient to start raising the drawbridge from the horizontal position, calculate:

i) the tension in the chain;

ii) the total force exerted on the pivot.

FIGURE 1: The Drawbridge Problem Statement.

Solution of the Drawbridge Problem

Following is a complete solution for the drawbridge problem. The solution has been separated sequentially into numbered steps, and is presented in Figures 2 to 9.

Additionally, it was possible to identify key relations in the steps of the solution. These are referred to as rules and have been identified on each of the solutions as a part of the solution step description.

TAE PHYSICS DRAWBRIDGE SOLUTION

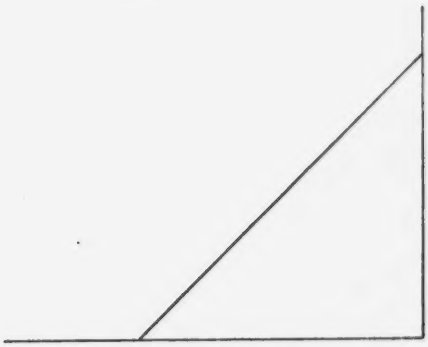
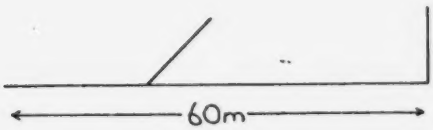
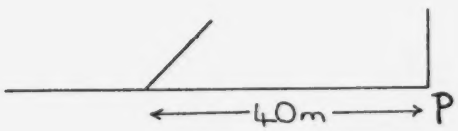
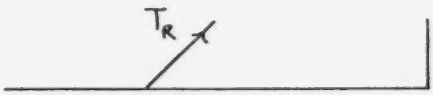
Reference Number	Solution Step	Step Description
1		Overall diagram with no distances or other markings.
2		Marking of drawbridge length.
3		Marking of chain contact at 40 m from the pivot P.
4		Marking of chain tension vector direction, with respect to the bridge.
5		Label to indicate T_R, the resultant chain tension required.

Figure 2: Drawbridge Solution Steps 1 to 5

TAE PHYSICS DRAWBRIDGE SOLUTION

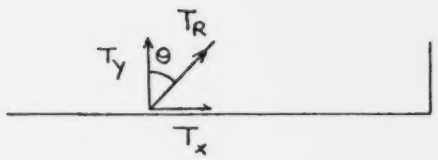
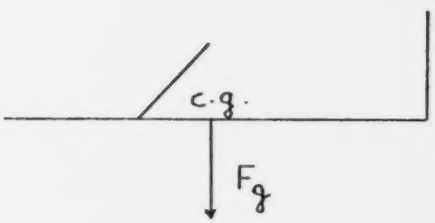
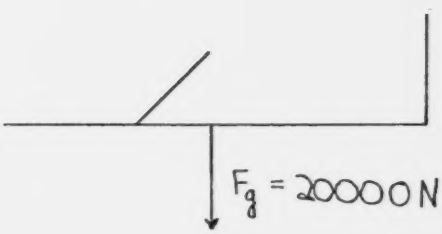
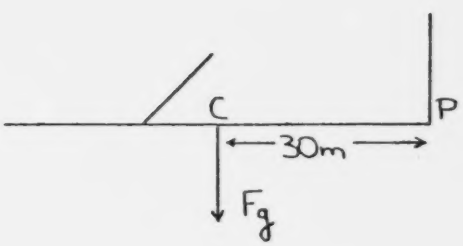
Reference Number	Solution Step	Step Description
6		Resolution of T_R into its vertical and horizontal components, T_y and T_x (or equivalent).
7	$T_y = T_R \cos\theta$	Equation of trigonometric relation between the resultant chain tension T_R and its vertical component T_y (or equivalent). RULE 1.
8		Recognition of the position, centre of gravity or weight vector in the uniform drawbridge.
9		Labelling of the value of the drawbridge weight.
10		Location of the drawbridge centre of gravity C, 30 m from the pivot P, i.e., in the centre of the bridge. RULE 2.

Figure 3: Drawbridge Solution Steps 6 to 10.

TAE PHYSICS DRAWBRIDGE SOLUTION

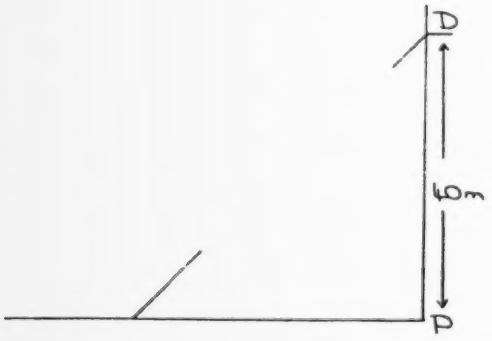
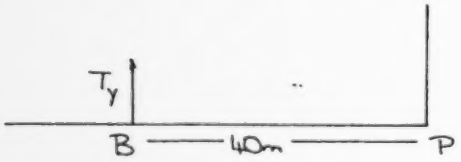
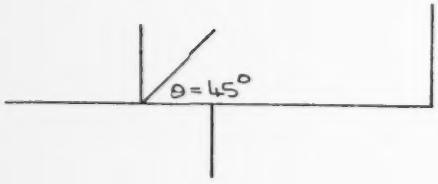
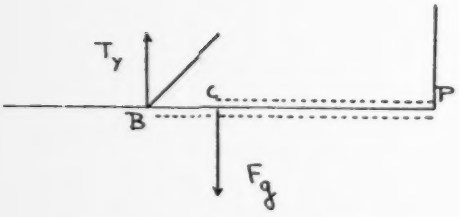
Reference Number	Solution Step	Step Description
11		Labelling of the height at which the drawbridge chain makes contact D, with the castle wall.
12	 $PB = PD = 40m$	Notation that height and horizontal distances of chain contact are equally distant from pivot P.
13		Recognition that the chain angle must be 45° to the drawbridge.
14	 $T_y \times BP = F_g \times CP$	Considering moments about P in the diagram leads to the statement of an equation of moments. RULE 3.

Figure 4: Drawbridge Solutions Steps 11 to 14

TAE PHYSICS DRAWBRIDGE SOLUTION

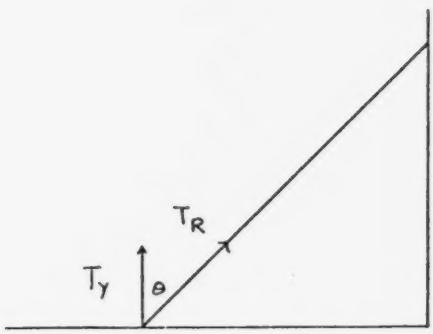
Reference Number	Solution Step	Step Description
15	$T_y \times 40 = 2.10^4 \times 30$	Substitution of numerical values in the equation of moments.
16	$T_y = \frac{2.10^4 \times 30}{40}$	Isolation of the unknown variable by cross multiplication.
17	$T_y = 1{\cdot}5 \times 10^4 N$	Simplification to find a value for the vertical component of chain tension T_y.
18		From the diagram is developed a trigonometric relation between known values and the required resultant chain tension T_R.
19	$T_R = \frac{T_y}{\cos\theta} = \frac{1{\cdot}5 \times 10^4}{\frac{1}{\sqrt{2}}}$	Equation of trigonometrical relation is followed by substitution and simplification leading to the value for chain tension as required in the problem statement.
20	$T_R = 1{\cdot}5 \times 10^4 \times \sqrt{2}$ $= 2{\cdot}12 \times 10^4 N$	

Figure 5: Drawbridge Solutions Steps 15 to 20

TAE PHYSICS DRAWBRIDGE SOLUTION

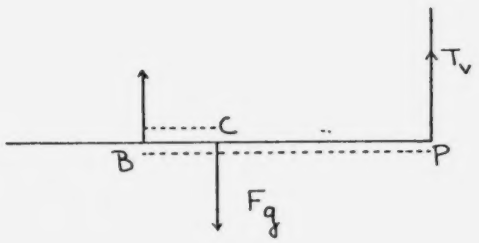
Reference Number	Solution Step	Step Description
21-29		These steps were considered adequately covered by steps 14-20. (Includes RULE 4.)
30		Consider moments about B where T_v (or its equivalent) is the vertical force on the pivot at point P.
31	$T_v \times BP = F_g \times BC$	Statement of the equation of moments about B. RULE 5.
32	$T_v \times 40 = 2.10^4 \times 10$	Substitution of numerical values into the equation of moments.
33	$T_v = \frac{2.10^4 \times 10}{40}$	Isolation of the vertical component of pivot force T_v by cross multiplication.
34	$T_v = 5000\ N$	Simplification to find the value of T_v, the vertical component of pivot force at P.

Figure 6: Drawbridge Solution Steps 30 to 34

TAE PHYSICS DRAWBRIDGE SOLUTION

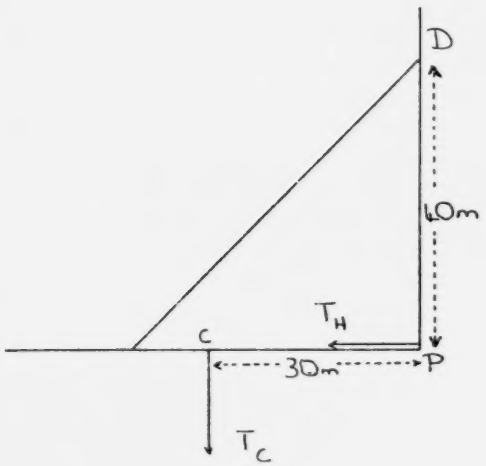
Reference Number	Solution Step	Step Description
35		Consider moments about D in order to find the value of the horizontal component T_H of pivot force at P.
36	$T_c \times PC = T_H \times DP$	Statement of the equation of moments about D. RULE 6.
37	$2.10^4 \times 30 = T_H \times 40$	Substitution of numerical values into the equation of moments.
38	$\frac{2.10^4 \times 30}{40} = T_H$	Isolation of the unknown variable T_H by cross multiplication.
39	$T_H = 1.5 \times 10^4 N$	Simplification to find the value of T_H, the horizontal component of compression at P.

Figure 7: Drawbridge Solution Steps 35 to 39

TAE PHYSICS DRAWBRIDGE SOLUTION

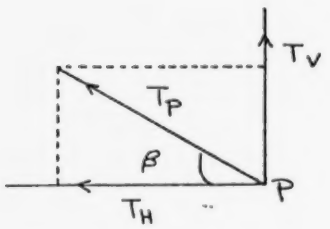
Reference Number	Solution Step	Step Description
39	$T_R \cos 45 = T_H = 1.5 \times 10^4 N$	(Alternative method) Equating of horizontal forces on drawbridge to find T_H.
		From the diagram is developed an equation for calculating direction of the resultant pivot force at P. RULE 7.
40	$\tan \beta = \frac{T_V}{T_H} = \frac{5000}{15000}$	Statement of trigonometric ratio and substitution of values therein, for direction of force at pivot.
41	$\beta = \tan^{-1} \frac{1}{3} = 18.43^\circ$	Simplification to find the angle of direction for resultant pivot force.

Figure 8: Drawbridge Solution Steps 39 to 41

TAE PHYSICS DRAWBRIDGE SOLUTION

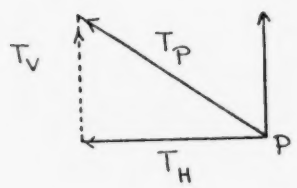
Reference Number	Solution Step	Step Description
42		Application of Pythagoras' Rule to the diagram about the pivot in order to calculate the resultant magnitude of pivot force T_p. RULE 8.
43	$T_P = \sqrt{(T_V)^2 + (T_H)^2}$	Statement of Pythagorean equation.
44	$T_P = \sqrt{(5000)^2 + (1.5 \times 10^4)^2}$	Substitution of numerical value into equation.
45	$T_P = 1.58 \times 10^4 N$ $T = 1.58 \times 10^4 N$ at an angle of 18.43° to the horizontal.	Simplification of values to provide an answer for the value of resultant force at the pivot, as required in the problem statement.

Figure 9: Drawbridge Solution Steps 42 to 45

THE JIBCRANE PROBLEM AND ITS SOLUTION

Reasons for selecting the jibcrane problem have already been detailed under the section on problem selection criteria.

The Problem Statement

The diagram represents a jibcrane. What are the forces in the jib and tie when the load is one tonne (10^3 kg)?

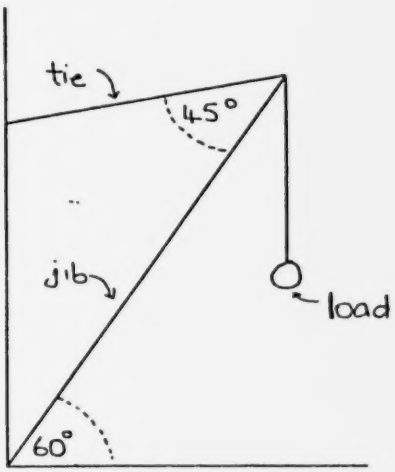


Figure 10: Diagram of Jibcrane

Solution to the Jibcrane Problem

The jibcrane problem presented in Figure 10 has several solutions. The four different solutions dealt with are referred to as follows:

Method 1 : Vector Component Analysis
Method 2 : Moment Equilibrium Analysis
Method 3 : Vector Triangle Calculation
Method 4 : Scaled Triangle Construction

Following is a complete solution for each of the four methods used by students to solve the jibcrane problem. Each solution method has been separated sequentially into numbered steps. As with the drawbridge solution, the numbered steps provide a cross reference to related tabulations and their graphs. This permits quick interpretations of graph trends and details.

TAE PHYSICS JIBCRANE SOLUTION

SOLUTION METHOD 1 : VECTOR COMPONENT ANALYSIS

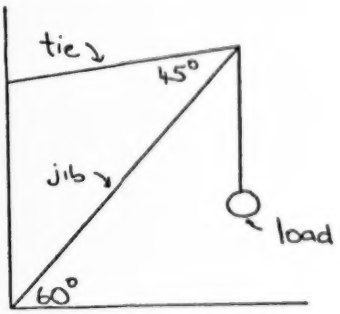
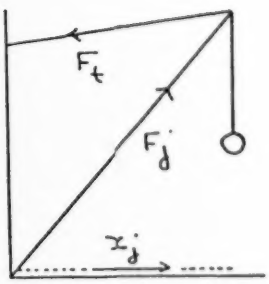
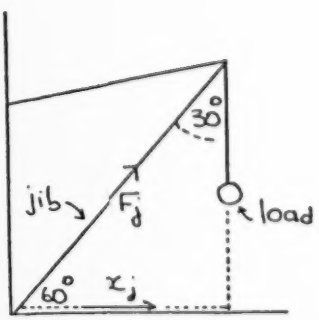
Reference Number	Solution Step	Step Description
1		Crane diagram (as given in the problem statement). Redrawing of diagram by examinee.
2		Identifying the tie force by symbol F_t (or equivalent) and by vector direction arrowhead.
3		Identifying the jib force by symbol F_j (or equivalent) and vector direction arrowhead.
4		Drawing and labelling the horizontal component of jib force, X_J.
5		Recognition of the size of the angle between jib and load as being 30°. Incorporating that angle into a trigonometric relation between jib force F_J and its horizontal component X_j.

Figure 11: Jibcrane Solution Steps 1 to 5 in Method 1

SOLUTION METHOD 1 : VECTOR COMPONENT ANALYSIS

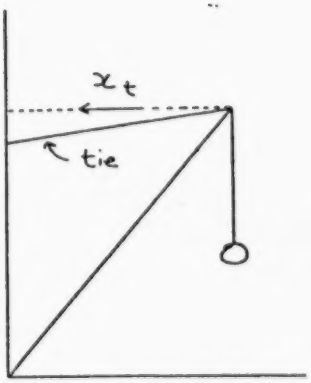
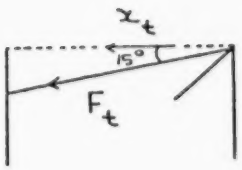
Reference Number	Solution Step	Step Description
	$\sin 30^\circ = \frac{x_j}{F_j}$	Statement of trigonometric relation between F_j and X_j (or equivalent with complementary angle). RULE 1.
6	$x_j = F_j \cdot \sin 30^\circ$	Cross multiplication and simplification to find a value for X_j the horizontal component of the jib force.
7		Drawing and labelling the tie force horizontal component X_t.
		Recognition that the angle size between the tie force F_t and its horizontal component X_t is 15°.
8	$\cos 15^\circ = \frac{x_t}{F_t}$	Statement of the trigonometric relation which will help find the value of X_t. RULE 2.

Figure 12: Jibcrane Solution Steps 6 to 8 in Method 1.

SOLUTION METHOD 1 : VECTOR COMPONENT ANALYSIS

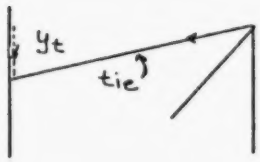
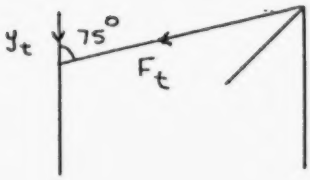
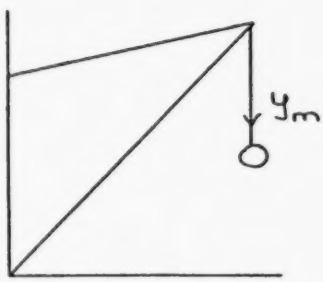
Reference Number	Solution Step	Step Description
9	$X_t = F_t \cos 15^\circ$	Cross multiplication and simplification to find a value for X_t.
10		Identification of Y_t, the vertical component of the tie force.
		Recognition of the angle size between F_t and Y_t as 75°.
11	$\cos 75^\circ = \frac{y_t}{F_t}$	Statement of the trigonometric relation between F_t and Y_t. RULE 3.
12	$y_t = F_t \cdot \cos 75^\circ$	Cross multiplication and simplification to find a value for Y_t.
13		Labelling vector direction of the load and its symbol Y_m.

Figure 13: Jibcrane Solution Steps 9 to 13 in Method 1.

SOLUTION METHOD 1 : VECTOR COMPONENT ANALYSIS

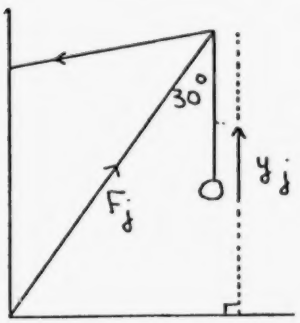
Reference Number	Solution Step	Step Description
14	$Y_m = mg$	Statement of the equation to find Y_m. RULE 4.
15	$Y_m = 10^3 \times 10 = 10^4 N$	Substitution and simplification.
16		Drawing and labelling of Y_j the vertical component of the jib force.
17	$\cos 30^\circ = \frac{Y_j}{F_j}$	Statement of trigonometric relation between the jib force F_j and its vertical component Y_j. RULE 5.
18	$Y_j = F_j \cdot \cos 30^\circ$	Cross multiplication and simplification to find a value for Y_j.
19	$\Sigma F_H = 0$	Statement of the law for equilibrium for horizontal components.

Figure 14: Jibcrane Solution Steps 14 to 19 in Method 1.

SOLUTION METHOD 1 : VECTOR COMPONENT ANALYSIS

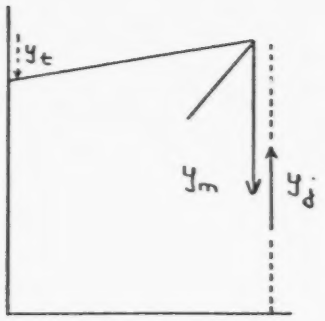
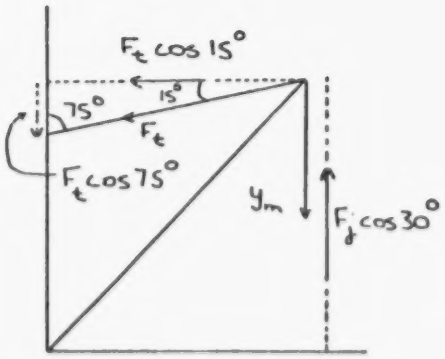
Reference Number	Solution Step	Step Description
20	$\therefore x_j = x_t$	Equation of equilibrium. RULE 6.
21	$F_j \sin 30^\circ = F_t \cos 15^\circ$	Substitution in equation.
22	$F_j = F_t \frac{\cos 15^\circ}{\sin 30^\circ} = F_t (1{\cdot}9318)$	Cross multiplication and simplification.
		Recognition of the equation of equilibrium for vertical components. RULE 7. $\Sigma F_v = 0$. Labelling of load vector Y_m, vertical component vector of jib Y_j, and vertical component vector of the tie Y_t.
23	$y_t + y_m = y_j$	Statement of the equation of equilibrium for vertical components.
		Labelling of component values. Substitution for Y_t and Y_j on diagram.

Figure 15: Jibcrane Solution Steps 20 to 23 in Method 1.

SOLUTION METHOD 1 : VECTOR COMPONENT ANALYSIS

Reference Number	Solution Step	Step Description
24	$F_t \cos 75^\circ + 10^4 = F_j \cos 30^\circ$	Substitution of component values and variables into the equation of equilibrium.
25	$F_j = F_t (1.9318)$	(From Step 22.) Substitution for jib force F_j in Step 24.
26	$\therefore F_t = 7.072 \times 10^3$ N	
27	$F_t \simeq 7.1 \times 10^3$ N	Isolation, simplification.
28	$F_j = F_t (1.9318)$	(From Step 22.) Substitution for F_j.
29	$F_j = 7072 \times 1.9318$	Substitution for F_t in order to find a value for F_j.
30	$F_j \simeq 13.7 \times 10^3$ N	Simplification and rounding off to produce a final answer for the jib force.

Figure 16: Jibcrane Solution Steps 24 to 30 in Method 1.

TAE PHYSICS JIBCRANE SOLUTION

SOLUTION METHOD 2 : MOMENT EQUILIBRIUM ANALYSIS

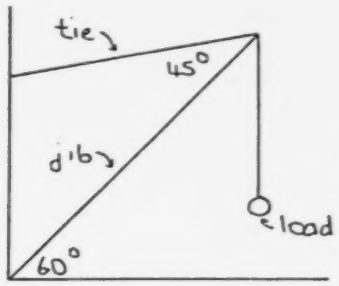
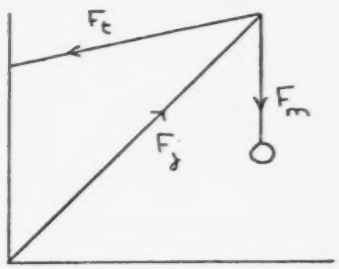
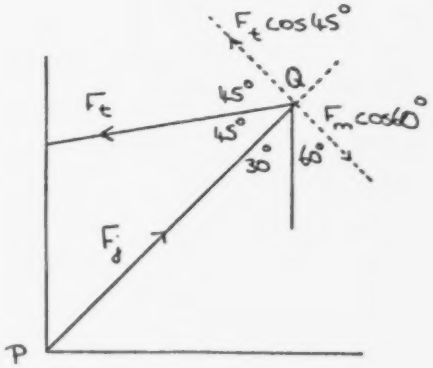
Reference Number	Solution Step	Step Description
1		Crane diagram (as given in the problem statement)
2		Identification of tie vector F_t, jib vector F_j and weight vector F_m by way of symbols and arrowheads. RULE 1.
3	$\Sigma CWM = \Sigma ACWM$	Statement of moments about any fulcrum. RULE 2.
		Orthogonal resolution perpendicular to the jib direction.

Figure 17: Jibcrane Solution Steps 1 to 3 in Method 2

SOLUTION METHOD 2 : MOMENT EQUILIBRIUM ANALYSIS

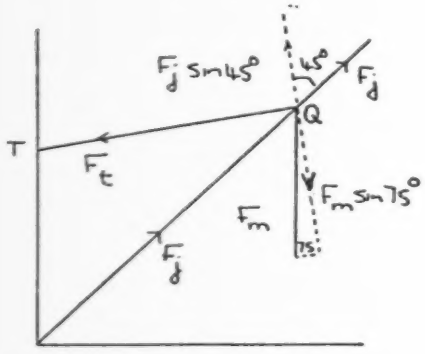
Reference Number	Solution Step	Step Description
4	$(F_t \cos 45^\circ)PQ = (F_m \cos 60^\circ)PQ$ $F_t \cos 45^\circ = F_m \cos 60^\circ$ $F_t = F_m \dfrac{\cos 60^\circ}{\cos 45^\circ}$	Statement of the equation of moments about the pivot P. RULE 3.
5	$= \dfrac{1000 \cdot 10 \cdot 0.5}{0.7071}$ $= 7070\ N$	Cross multiplication, substitution and simplification to find tie force F_t.
6		Orthogonal resolution perpendicular to tie direction.
7	$\Sigma CWM = \Sigma ACWM$ $(F_j \sin 45^\circ)TQ = (F_m \sin 75^\circ)TQ$	Statement of equation of moments about T. RULE 4.
8	$F_j \sin 45^\circ = F_m \sin 75^\circ$	Substitution in equation of moments about T.
9	$F_j = \dfrac{1000 \cdot 10 \cdot 0.9659}{0.7071}$ $= 1.366 \times 10^4\ N$	Cross multiplication and simplification to find an answer for F_j.

Figure 18: Jibcrane Solution Steps 4 to 9 in Method 2.

TAE PHYSICS JIBCRANE SOLUTION

SOLUTION METHOD 3 : SOLUTION OF TRIANGLES CALCULATION

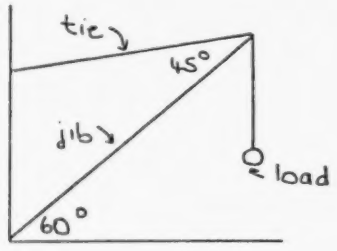
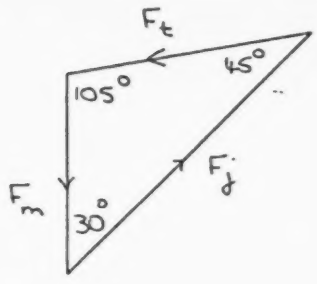
Reference Number	Solution Step	Step Description
1		Crane diagram (as given in the problem statement).
2		Representation of the three vectors in a triangle of equilibrium with identified vectors and angles.
3	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Statement of general trigonometric relationship.
4	$\frac{F_t}{\sin 30} = \frac{F_m}{\sin 45}$	Substitution of numerical values to find F_t.
	$F_t = \frac{10^4 \cdot \sin 30}{\sin 45}$	Cross multiplication to isolate F_t.
5	$F_t = 7071\ N$	Simplification and answer for the tie force.
6	$\frac{F_j}{\sin 105} = \frac{F_m}{\sin 45}$	Substitution to find F_j.
7	$F_j = \frac{10^4 \sin 105}{\sin 45} = 13660\ N$	Cross multiplication and answer for jib force.

Figure 19: Jibcrane Solution Steps 1 to 7 in Method 3.

TAE PHYSICS JIBCRANE SOLUTION

SOLUTION METHOD 4 : SCALE DIAGRAM CONSTRUCTION

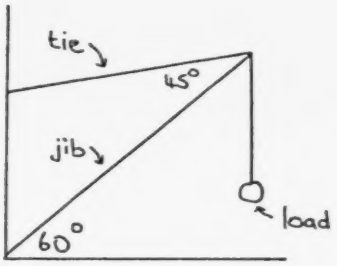
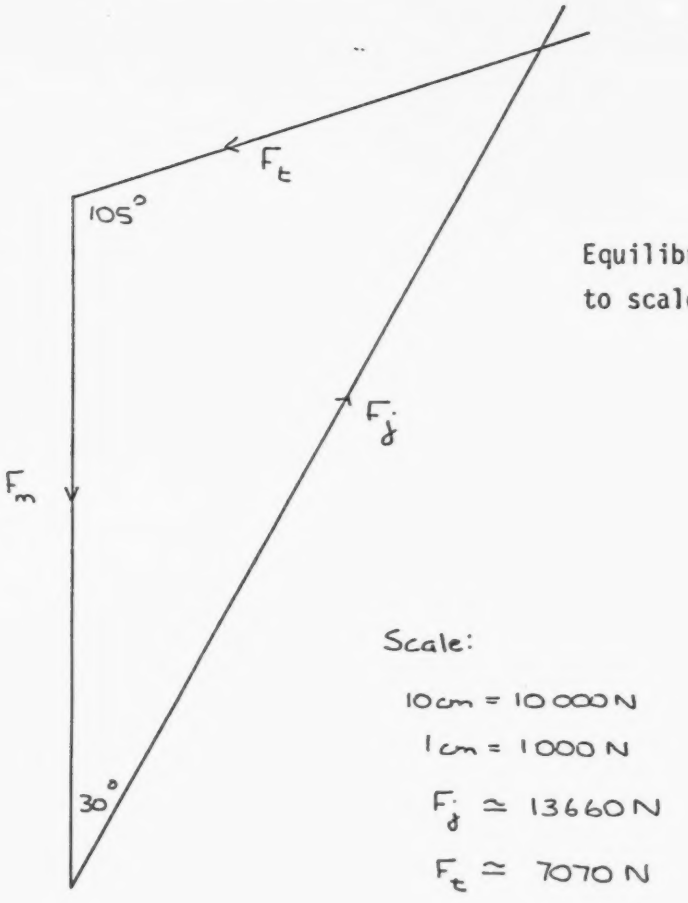
Reference Number	Solution Step	Step Description
1		Crane diagram (as given in the problem statement).
2		Equilibrium triangle drawn to scale. RULE 1.

Figure 20: Jibcrane Solution Steps 1 to 2 in Method 4.

DESCRIPTION OF THE FOLLOW-UP STUDY TEST INSTRUMENT

The instrument used for the Mechanics Vector Analysis Test consisted of thirty different diagrams on five pages (Appendix A).

The test was made up of five sections. These were used to measure the ability of students:

1. to sketch the resultant of two vectors.

2. to identify sets of forces in equilibrium and sets of forces not in equilibrium.

3. to insert with an arrowhead, the correct direction of vectors on a freebody diagram in equilibrium, where one out of the three directions had already been nominated.

4. to describe the physical principles used. This work was considered extraneous to the thesis and was therefore omitted from analysis.

5. to draw the directions and the point of concurrence, for three forces acting on an isolated body, to keep it in equilibrium.

In all, there were 106 subsectional steps. Each step was marked independently, according to a specifically developed marking key, discussed in detail further on (see Appendix A, page 195).

The results of this test are discussed in the latter part of Chapter 4. A copy of the Mechanics Vector Analysis Test may be found in Appendix A.

Construction of Mechanics Vector Analysis Test

Selection of test items was based upon the observed difficulties that students experienced in handling vectors associated with the drawbridge and jibcrane solutions. These difficulties were related to vectors involving equilibrium of forces in free body diagrams.

Validity and Reliability of Test Items

Validity of the Mechanics Vector Analysis test was checked as suitable, by consulting practicing physics teachers and the TAE physics examiner. The test was considered appropriate for the current physics syllabus.

Reliability of the test was calculated using Cronbach's coefficient alpha, for an estimate of internal consistency. An item score matrix listed sixty item scores for each of 160 examinees, with scores of one for right answers and zero for wrong answers. Further details obtained from the analysis are recorded in Appendix B.

Values of alpha coefficients were: for examinees, 0·83 and, for items, 0·84. The coefficients obtained from this analysis indicate a high level of reliability for the Mechanics Vector Analysis test.

SETTING UP SOLUTION METHODS

Each step of a model solution should be justifiable as an essential stage in the logical explanation of just how the solution was planned and the answer attained. Just how many of the steps in a model solution may be justifiably omitted under examination conditions, really depends upon personal judgement as to what constitutes an "acceptable" level of explanatory detail or logical development.

For example, a solution which demanded a complete written argument, listing all the developmental steps of mathematical logic, would require many times the number of steps judged by the author as necessary to arrive a correct answer to each of the drawbridge and jibcrane problems. The set of solution steps necessary therefore, would vary according to the judgement of different problem solvers. The condition of time constraint is a very real difficulty in a solution sequence where there are many possible solution steps. When several of these steps can be handled mentally, it is often very reasonable for the problem solver to omit them from the written record of solution or to summarise their essential features within a diagrammatic translation and in substituted equations. This situation occurred quite frequently in student solution attempts at the selected problems.

The examiner's marking key for the drawbridge problem provided only skeletal guidance to key solution steps. It therefore became a matter of author's judgement on what steps should provide assistance in evaluating student performance against a checklist of model solution steps. The decision made was to record, as completely as possible, every step of physics-based logic deemed useful in clarifying and linking the problem statement with its final answer(s) for each of the problems considered. This included statements of physics principles, introductory statements of procedure, as well as several labelled diagrams designed to assist visualisation of explanatory steps and calculation.

The purpose of recording every possible step of physics-based explanatory logic, had several uses. It was realised that no student could be expected to record every step of the proposed model

solution in the examination time allocated (7 minutes). It therefore became more appropriate to analyse those steps which a student had written down, and to record as omissions, any steps which had been skipped. This would then provide some consistency and guidance in recording and coding data from each student solution.

There were no distinctly differing solution methods for the difficult drawbridge problem. Perhaps the sample of 700 papers from a population of 2518 physics examinees was not large enough to include, by random sampling, those minor alternate methods. This situation contrasts sharply with the four distinctly different methods identified in the whole population analysis conducted on student solutions to the jibcrane problem.

Skipped steps in a student solution were referred to as "omissions". The recording of these "omissions" had several aims. The first aim was to provide a data record of student-based solution priorities, where time was a critical limitation upon a student's presentation of detailed explanatory logic. The second aim was to have a data record of actual student performance for comparison with what had been set up as the detailed model of a complete solution statement. This would then permit analytical judgement of essential steps omitted, to be separated from those non-essential solution steps. Finally, such a record of student performance would help identify common student omissions as well as common student errors.

Details of steps provided in the model drawbridge and jibcrane solutions are supported by the following background information.

Since the TAE claims to be attempting examination of the understanding of physics principles and their application (TAE Syllabus, 1978), it was appropriate to adopt these claims as essential criteria in arriving at decisions on solution sequences and steps. Within this context, it was considered relevant to record step statements identifying specific rules which were being applied, prior to a statement of their mathematical translation and substitution of values therein.

As part of the step sequences in the drawbridge and jibcrane solutions, the applied rules of physics have been labelled as such, as part of the step description. The purpose of noting rules as the solution sequence proceeded, was to provide a record of the number of rules needing to be applied before reaching the final answer. Their record also marks particular stages arrived at by student problem solvers in their solution attempts. Finally, this data record could be more readily cross-referenced with the tabulations of error occurrence and also with the related graphical translation of error occurrence, as presented in Chapter 4.

LIMITATIONS OF THE STUDY

The following were the limitations of the study.

1. Only a very limited area of mechanics problem solving could be examined.

2. Because of choice within each paper, a fully representative sample of student problem solving ability was not guaranteed.

3. Time limitations and examination conditions tend to affect student problem solving performance. This performance will probably differ from that where time is not limited and performance is not rushed. In such time-constrained circumstances, it may be impractical for students to record all the desirable steps in a logical explanation.

4. Correct solutions and answers may have been dependent on speed of reasoning. Under time constrained conditions, a slower problem solver may not have been fairly represented in this analysis.

5. Of the many areas of student difficulty found in solving the drawbridge and jibcrane problems, follow-up research was carried out on only one common difficulty, namely, diagrams and their use in solving problems.

6. The two examination problems chosen for investigation were regarded by the examiners as of more than average difficulty. So, in generalising the results of analysis from these problems, it should be noted that their degree of difficulty was designed to be beyond the ability of the average physics examinee. This therefore, becomes a limiting factor in the overall evaluation of student competence in physics problem solving.

7. Specific limitations are placed upon the quality of generality for the conclusions and recommendations of this study. The findings and the recommendations are closely concerned with

physics problems wherein diagrams can be used. The second limitation springs from the restriction of this study to mechanics problems involving bodies in equilibrium under the action of three forces. Application of diagrammatic analysis may not become as appropriate in other fields of physics problem solving.

Scoring of Mechanics Problem Scripts

Associated with problem selection, was the need to develop a step scoring process, within which student solution attempts could be scaled according to their proximity to a correct statement of fact, formula, figures, or diagrammatic representation.

Development of the Solution Scoring Scale

Specifically developed for this study, was a four point scoring scale, designed to distinguish two levels of omission and a 'right' or 'wrong' statement for every solution step in each student attempt.

This scoring scale recorded every student solution step approaching the final answer. It was also sufficiently detailed to specify an instantaneous step score as well as to record a general trend or the occurrence of an obstacle to solution progress and follows in Table 2.

TABLE 2

Coding Translation for Scoring Scale

0	No response (step not attempted).
1	Step unstated (step was apparently deliberately skipped).
2	Step stated incorrectly.
3	Step stated correctly.

This four point limit of the scoring scale was thought to be sufficiently flexible for extra scales to be added or a different scale to be developed at a later stage.

As had previously been allowed for, the steps in visual translation of a problem were able to be selected out. They were then dissected for error analysis. From this experience, verification test questions were incorporated in the Mechanics Vector Analysis Test. This classroom testing programme concentrated solely on visualisation performance. From experience of the initial scoring process, the same scoring scale remained thoroughly appropriate, provided it was related to a specialised diagram description system.

This diagram descriptive analysis was adjusted to a need for precise scoring instructions in a freebody diagram, in vector marking order and in the scaling mechanism of the answer presented.

TABLE 3

Mechanics Vector Analysis Test

Marking Key for Visualisation

Performance Scoring Scale

0 No response.

1 Step omitted.

2 Step recorded incorrectly.

3 Step recorded correctly.

The marking key above was applied to each of the eight items of diagram analysis in Table 4 below.

TABLE 4

Mechanics Vector Analysis Test

Marking Order for Pages 1, 2, 3

1 Point of application of the force.

2 Line of action of the force.

3 Magnitude of the vector.

4 Direction of the vector.

Marking Order for Page 5

1 Point of application of each force.

2 Line of action of each force.

3 Direction of each vector.

4 Point of triple intersection for each vector.

Every essential feature of each vector diagram had to be marked separately and tabulated in ordered groups for statistical comparative analysis. Using the appropriate marking order, in conjunction with the visualisation performance scale, greatly simplified the multiple marking difficulties, by providing a more organised marking procedure.

Mechanics Vector Analysis Test

The design and purpose of the vector analysis test have been detailed in this Chapter. Development and discussion of the relevant marking keys were also presented here, as an appropriate preview to the data record and data analysis that are presented later. At this point, it is appropriate to review the intentions of the follow-up research prior to analysis and interpretation of results.

Both exploratory research and population analysis have identified the most common student difficulty as diagrammatic visualisation of each problem and its solution. It was found however, that neither of these research phases had given proper recognition to the detailed record required, for complete analysis of error types involved in diagrammatic visualisation. This lack of focus upon the target research topic became evident only as a result of completing and comparing phase one and phase two.

Consequent upon this finding, the Mechanics Vector Analysis Test was given the task of detailed identification of error types in student performance on diagrammatic visualisation. The refinement of detail now required, brought this follow-up phase to the point of being the central purpose of the entire study.

Design of the follow-up research was closely related to TAE student competence. To this end, arrangements were made to visit four schools involving seven class groups, each containing about 25 Year 12 physics students. The timing was organised to be about six weeks prior to their 1981 Tertiary Admission Examination. This, it was felt, would permit most students to be in a proper state of readiness for the testing of diagram analytic ability.

CHAPTER 4

RESULTS AND DISCUSSION

Previous chapters have dealt with the concern of the study, a literature review and discussion of some terminology in physics problem solving. Chapter 3 raised questions for research and presented hypotheses for investigation. Design and methodology for the study were also outlined.

RESEARCH METHOD AND DATA ANALYSIS

Introduction

As alluded to under the statement of hypotheses in Chapter 3, one of the aims of this study was to identify common difficulties encountered by tertiary admission examinees in physics problem solving. This chapter presents the results of that analysis.

Investigation of two solution attempts for TAE physics problems was carried out. Analysis involved the separation of each step in every student solution script into a categorised error code. From this tabulation, was compiled a record of errors and their frequency, for each step of every solution method. Details of errors, marking codes and solution methods have been presented in Chapter 3 as part of the research design and methodology.

DRAWBRIDGE SOLUTION

From the sample of 700 scripts used, 416 student papers contained solution attempts to the drawbridge problem.

RESULTS OF THE DRAWBRIDGE SOLUTION ANALYSIS

General analysis of the drawbridge solution results was commenced by tabulating the error frequency occurrence for each solution attempt, in Table 5 below. Errors of omission and of incorrect step statement for the drawbridge solution part (i) are summarised in the following table.

Many of the drawbridge solution scripts showed only part (i) completed and part (ii) not attempted at all. Because of this, and also because of the length of the complete solution, it was decided to restrict the initial exploratory investigation to part (i) of the drawbridge solution.

TABLE 5

Summation of errors in the drawbridge solution scripts part (i) and the number of students with that error frequency. (N = 416) (error codes: 0, 1, 2)

Error Total	Student Number	Error Total	Student Number
0	1	11	51
1	5	12	62
2	16	13	55
3	25	14	40
4	18	15	23
5	4	16	6
6	7	17	3
7	11	18	4
8	14	19	3
9	32	20	1
10	35		

Table 5, recording the frequency of error occurrence, forms the basic reference for graphs and other tables throughout this study.

From Table 5, in conjunction with Figure 21, a decision was made by the author to separate drawbridge solution scripts into levels of performance ability. This separation of ability levels was based upon the number of errors occurring.

The following graph (Figure 21) provides the table of error occurrence in part (i) for the 416 attempted solutions to the drawbridge problem.

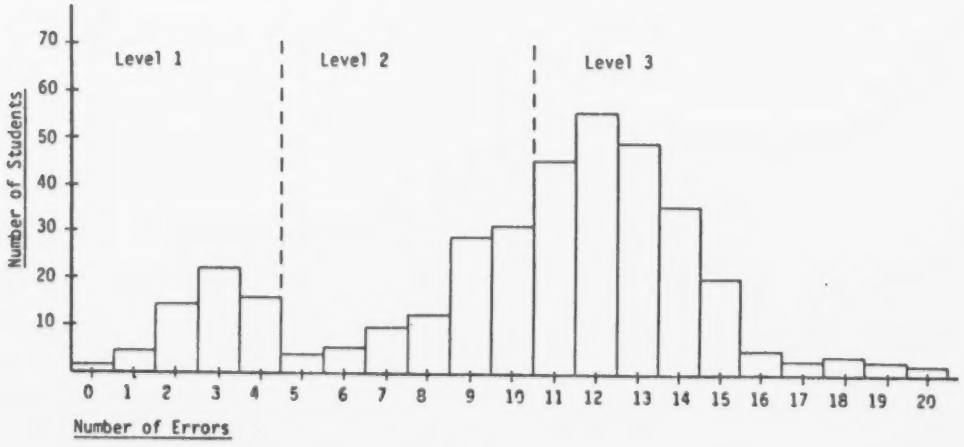


Figure 21

Summation of Errors (Coded 0, 1, 2) in the Drawbridge Solution Scripts Part (i) and the Number of Students with that Error Frequency

The graph was used to form three arbitrary subgroups, based on the number of errors in their solution process, for Part (i) of the drawbridge problem. These results are presented in Table 6.

Table 6

Separation of student solution performance by means of error frequency using part (i) of the drawbridge solution scripts (errors coded 0, 1, 2)

Subgroup	Error Frequency	Number of Students
Level 1	0 - 4 errors	65
Level 2	5 - 10 errors	103
Level 3	> 10 errors	248
Sample size		416

Decision on the arbitrary subdivision of ability into levels of problem solving performance was based upon error frequency counts. The boundaries of each level were not regarded as a critical decision. They were regarded more as a means of rough separation between the "correct", the "partially correct" and the "unsuccessful" solution attempts. One of the reasons for this separation was to remove the "unsuccessful" solution scripts from further analysis in this study.

At the frequency of > 10 errors, the author assessed that there ceased to be evidence of any organised pattern of attack or method in solving the drawbridge problem. For this reason, a total of more than 10 errors in each solution was chosen as the cutoff point between the Level 2 partially correct solution scripts and the Level 3 unsuccessful solutions.

Students of Level 1 had made only minor omissions or incorrect statements. These students were judged to have obtained a correct or virtually correct solution to the drawbridge problem. Students of Level 2 were partially correct and had demonstrated some form of method in the solution process. Level 3 student subgroup, numbering 248, were rejected prior to further error analysis. It was decided that at this level of performance, students had produced far too many errors for their scripts to be useful in the identification or classification of error types. In Levels 1 and 2, 168 solution attempts were retained for complete analysis. It represents the top 40% of student solution attempts to the drawbridge problem.

Analysis of errors for the drawbridge solution scripts is summarised in Tables 7 and 8. Each table includes an error code summary of Level 1, then Level 2 and finally, a composite analysis of Levels 1 and 2 combined. The error code translation for the following tables has been fully detailed in Chapter 3.

In each of the three subsections of these summary Tables, steps coded 1 and 3 have been combined to record the percentage of correct attempts for each step. Steps coded 0 and 2 have been combined to summarise the percentage of incorrect attempts for each step of the drawbridge solution.

From the error code summary in Tables 7 and 8, column graphs have been constructed and these are presented in Figures 22 and 23. These graphical analyses permit a comparison and contrast between problem solving performances of Level 1 and Level 2. The graphs show at a glance, where major common difficulties have occurred.

Table 7

Summary of Errors in the Drawbridge Solution Scripts Part (i) and the Number of Students with that Error Frequency

Step	LEVEL 1 : N = 65								LEVEL 2 : N = 103								COMPOSITE : N = 168							
	Error Codes				0&2	%	1&3	%	Error Codes				0&2	%	1&3	%	Error Codes				0&2	%	1&3	%
	0	1	2	3					0	1	2	3					0	1	2	3				
1	0	0	0	65	0	0·0	65	100	0	0	1	102	1	0·9	102	99·1	0	0	1	167	1	0·6	16	99·4
2	0	3	0	57	0	0·0	65	100	0	13	1	89	1	0·9	102	99·1	0	21	1	146	1	0·6	167	99·4
3	0	7	0	58	0	0·0	65	100	0	19	1	83	1	0·9	102	99·1	0	26	1	141	1	0·6	167	99·4
4	0	0	1	64	1	1·5	64	98·5	0	3	1	99	1	0·9	102	99·1	0	3	2	163	2	1·2	166	98·8
5	0	59	0	6	0	0·0	65	100	0	92	0	11	0	0·0	103	100	0	151	0	17	0	0·0	168	100
6	0	7	0	58	0	0·0	65	100	3	25	1	74	4	3·9	97	96·1	3	32	1	132	4	2·4	164	97·6
7	1	31	1	32	2	3·1	63	96·9	6	55	17	25	23	22·3	80	77·7	7	86	18	57	25	14·9	143	85·1
8	0	0	0	65	0	0·0	65	100	0	4	12	87	12	18·5	81	81·5	0	4	12	152	12	7·1	156	92·9
9	0	0	2	63	2	3·1	63	96·9	0	1	7	95	7	6·8	96	93·2	0	1	9	158	9	5·4	159	94·6
10	0	1	0	64	0	0·0	65	100	1	15	13	74	14	13·6	79	86·4	1	16	13	138	14	8·3	154	91·7
11	0	1	0	64	0	0·0	65	100	0	4	0	99	0	0·0	103	100	0	5	0	163	0	0.0	168	100
12	0	0	0	65	0	0·0	65	100	0	2	1	100	1	0·9	102	99·1	0	2	1	165	1	0.6	167	99·4
13	0	0	0	65	0	0·0	65	100	2	10	2	89	4	3·9	99	96·1	2	10	2	154	4	2.4	164	97.6
14	0	8	1	56	1	1·5	64	98·5	31	11	0	61	31	30·1	72	69·9	31	19	1	117	32	19.0	136	81·0
15	0	52	0	13	0	0·0	65	100	32	57	4	10	36	35·0	67	65·0	32	109	4	23	36	21.4	132	78·6
16	0	0	0	65	0	0·0	65	100	31	1	51	20	82	79·6	21	20·4	31	1	51	85	82	48·8	86	51.2
17	0	0	0	65	0	0·0	65	100	33	2	54	14	87	84·5	16	15·5	33	2	54	79	87	51.8	81	48.2
18	0	0	0	65	0	0·0	65	100	16	2	69	16	85	82·5	18	17.5	16	2	69	81	85	50·6	83	49·4
19	0	1	0	64	0	0·0	65	100	16	3	73	11	89	86·4	14	13.6	16	4	73	75	89	53·0	79	47·0
20	0	5	3	57	3	4·6	52	95·4	4	6	84	9	88	85·4	15	14·6	4	11	87	66	91	54·2	77	45·8

21 BLANK STEP TO SEPARATE PARTS (i) AND (ii)

Table 8

Summary of Errors in the Drawbridge Solution Scripts Part (ii) and the Number of Students with that Error Frequency

	LEVEL 1 : N = 65								LEVEL 2 : N = 72								COMPOSITE : N = 137							
Step	Error Codes 0	1	2	3	0&2	%	1&3	%	Error Codes 0	1	2	3	0&2	%	1&3	%	Error Codes 0	1	2	3	0&2	%	1&3	%
22	0	1	2	62	2	3·1	63	96·9	2	3	55	12	57	79·2	15	20·8	2	4	57	74	59	43·1	78	56·9
23	1	19	1	44	2	3·1	63	96·9	12	18	26	16	38	52·8	34	47·2	13	37	27	60	40	29·2	97	70·8
24	1	10	0	54	1	1·5	64	98·5	28	9	0	35	28	38·9	44	61·1	29	19	0	89	29	21·2	108	78·8
25	1	57	0	7	1	1·5	64	98·5	29	35	3	5	32	44·4	40	55·6	30	92	3	12	33	24·1	104	75·9
26	2	0	0	63	2	3·1	63	96·9	28	0	29	15	57	79·2	15	20·8	30	0	29	78	59	43·1	78	56·9
27	3	0	0	62	3	4·5	62	95·5	19	1	40	12	59	81·9	13	18·1	22	1	40	74	64	46·7	73	53·3
28	3	0	1	61	4	6·2	61	93·8	19	1	42	10	61	84·7	11	15·3	22	1	43	71	65	47·4	72	52·6
29	3	3	3	56	6	9·3	59	90·7	19	5	42	6	61	84·7	11	15·3	22	8	45	62	67	48·9	70	51·1
30	48	5	0	12	48	73·8	17	26·2	68	1	0	3	68	94·4	4	5·6	116	6	0	15	116	84·7	21	15·3
31	48	16	0	1	48	73·8	17	26·2	68	3	0	1	68	94·4	4	5·6	116	19	0	2	116	84·7	21	15·3
32	48	3	3	11	51	78·5	14	21·5	68	0	1	3	69	95·8	3	4·2	116	3	4	14	120	87·6	17	12·4
33	46	0	3	16	49	75·4	16	24·6	68	0	2	2	70	97·2	2	2·8	114	0	5	18	119	86·9	18	13·1
34	44	0	6	15	50	76·9	15	23·1	68	0	2	2	70	97·2	2	2·8	112	0	8	17	120	87·6	17	12·4
35	48	2	0	15	48	73·8	17	26·2	64	3	1	4	65	90·3	7	9·7	112	5	1	19	113	82·5	24	17·5
36	46	16	0	3	46	70·8	19	29·2	64	5	0	3	64	88·9	8	11·1	110	21	0	6	110	80·3	27	19·7
37	46	2	0	17	46	70·8	19	29·2	63	0	2	7	65	90·3	7	9·7	109	2	2	24	111	81·0	26	19·0
38	46	2	0	17	46	70·8	19	29·2	63	1	2	6	65	90·3	7	9·7	109	3	2	23	111	81·0	26	19·0
39	42	0	0	23	42	64·6	23	35·4	62	0	4	6	66	91·7	6	8·3	104	0	4	29	108	78·8	29	21·2
40	52	2	5	6	57	87·7	8	12·3	69	1	2	0	71	98·6	1	1·4	121	3	7	6	128	93·4	9	6·6
41	52	2	6	5	58	89·2	7	10·8	68	0	4	0	72	100	0	0	120	2	10	5	130	94·9	7	5·1
42	45	2	10	8	55	84·6	10	15·4	62	6	4	0	66	91·7	6	8·3	107	8	14	8	121	88·3	16	11·7
43	46	4	7	8	53	81·5	12	18·5	63	0	9	0	72	100	0	0	109	4	16	8	125	91·2	12	8·8
44	45	4	8	8	53	81·5	12	18·5	53	1	18	0	71	98·6	1	1·4	98	5	26	8	124	90·5	13	9·5
45	4	1	50	10	54	83·1	11	16·9	19	1	52	0	71	98·6	1	1·4	23	2	102	10	125	91·2	12	8·8
46	50	2	7	6	57	87·7	8	12·3	67	0	5	0	72	100	0	0	117	2	12	6	129	94·2	8	5·8

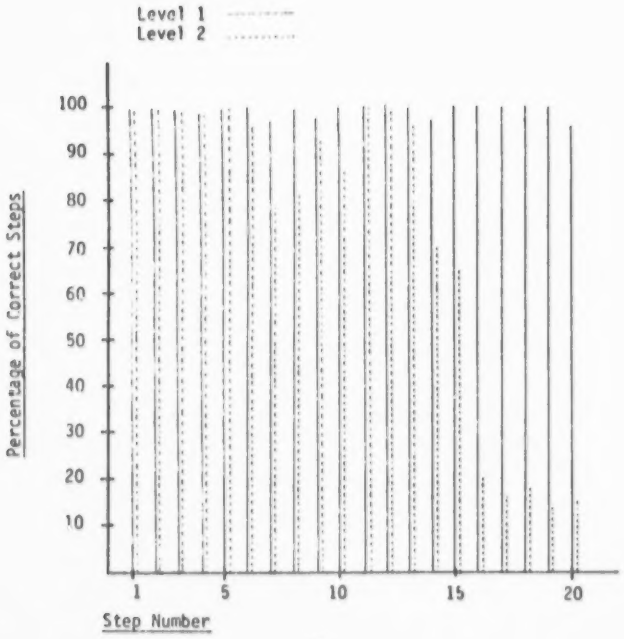


Figure 22

Drawbridge Solution Part (i)

Comparative Column Graphs for Levels 1 and 2
Percentage of Steps Completed Correctly

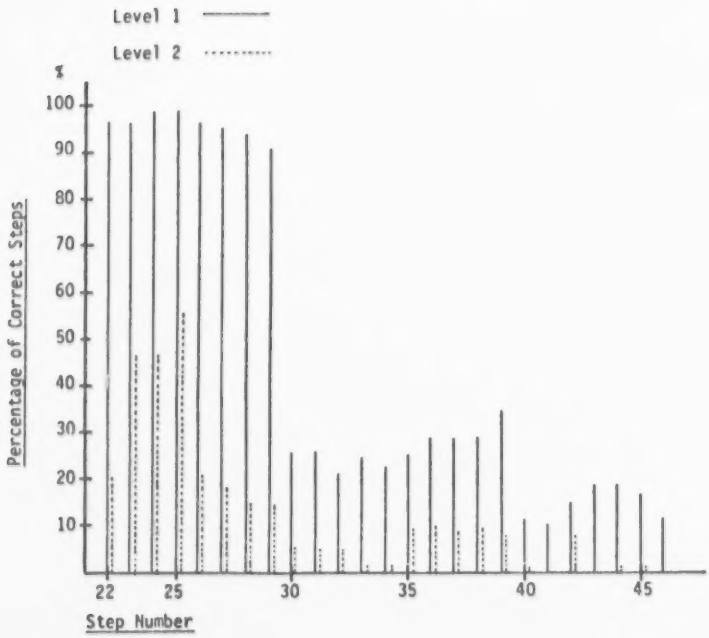


Figure 23

Drawbridge Solution Part (ii)

Comparative Column Graphs for Levels 1 and 2
Percentage of Steps Completed Correctly

Figures 22 and 23 present a summary of correctly completed statements for every step of the drawbridge solutions from Part (i) and (ii). The graphs give a picture of where major common obstacles occurred. They also provide a reference for discussion of common difficulties.

As was previously stated, the error frequency record for part (i) of the drawbridge solution became the basic reference for ability level separation. That error frequency itself however, needs discussion before further analysis proceeds.

The term "error", for codes 0, 1, 2 has included either the omission or incorrect statement of any solution step. The inclusion of errors of omission may not be an appropriate record of errors in student understanding or of problem processing. Such errors of omission may have been caused by the time constraints imposed under examination conditions. A correct subsequent step to any omitted step may suggest that the student understood and allowed for that step, even though it was not written down. Such omissions were given code 1.

Results of the errors by incorrect statement (error code 2) are presented below in Figure 24, to illustrate the steps which provided significant obstacles to a correct solution procedure.

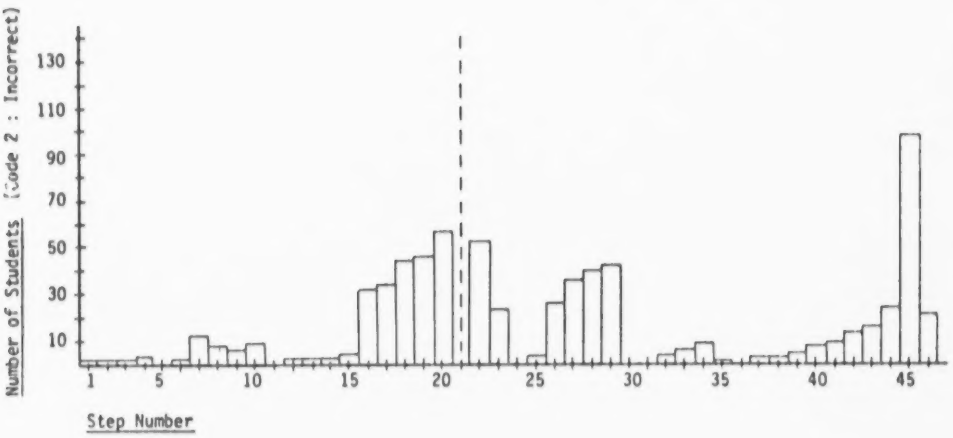


Figure 24

Summation of Code 2 Errors in the Drawbridge Solution Scripts.
Composite of Levels 1 and 2 for Parts (i) and (ii).
Code 2 indicates step stated incorrectly.

From Figure 24, it can be seen that Step 7 was an early obstacle for students. More erroneous statements (code 2 errors) occurred in Step 7 (Rule 1 explained in Chapter 3) than for all of the previous 6 steps combined. Step 7, or Rule 1, involved establishing a trigonometric relation between the resultant chain tension T_R and its vertical component T_y. (Detail in Chapter 3.)

The erroneous statements occurring in Step 7 may have arisen from, or may have been related to, the 11% (Level 1) and 24% (Level 2) of skipped steps of Step 6. Step 6 required the diagram analysis of the chain tension T_R into its vertical and horizontal components (Figure 25 below).

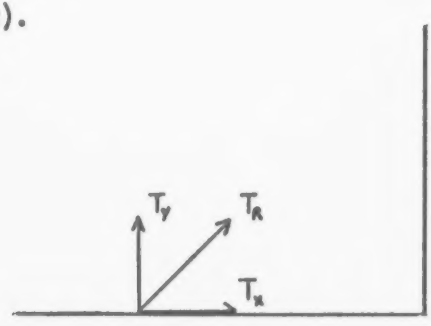


Figure 25: Drawbridge Solution Step 6.

The equation required in Step 7 was $T_Y = T_R \cos \theta$. Analysis of error data for this step will be expanded in the next section under Comparative Analysis of Levels 1 and 2. At this stage it will be noted however, that the equation $T_Y = T_R \cos \theta$ is a diagrammatic relationship. This means that its correct statement depended on component vector diagrams.

Discussion of the Results of the Error Analysis

From the composite summary of errors (page 87, Step 4), it may be seen that 97% of students gave evidence of seeing the need for a diagram to illustrate the forces on the chain. However, only 10% of solution attempts (Step 5) labelled this chain tension vector with a correct vector direction arrowhead and a suitable symbol such as T_R. The diagram analysis in Step 6, of the chain tension T_R, into its vertical component T_Y and its horizontal component T_X was correctly illustrated in 79% of the solution attempts being analysed.

At this stage of the solution process, only 34% of these students wrote down an equation of trigonometric relation between the resultant chain tension T_R and its vertical component T_Y in the form: $T_Y = T_R \cos \theta$ (see Step 7). In contrast, most solution attempts put direct numerically substituted values, into an equation for T_Y, in terms of T_R and $\cos \theta$. Of these substitutions, 49% of them were incorrect statements (see Step 19). This result illustrates the distinction, according to Larkin (1981), between novices and expert problem solvers.

> "Experienced physicists solve even simple problems in ways that are very different from the solutions produced by beginning students ... .
> .....

> [An expert proceeds through] a verbal statement of the problem ... [translating it to] a sketch of the physical situation [on through] a drawn freebody diagram [leading to] a set of equations constructed from the representation. ... An expert problem solver almost invariably shows some evidence of the two intermediate representations.
>
> .....
>
> In contrast, a novice solver commonly tries to construct equations directly from the verbal statement of the problem, and even more commonly skips the third or physical representation, trying to jump directly from a sketch of the real world situation to a set of equations."
>
> (Larkin, 1981, p. 535)

Discussion of these results will concentrate on student failure to provide a gradual and logical translation from the physical diagram to the ultimate aim, that of calculating a correct numerical value for T_R, the resultant chain tension.

That the diagram was incorrectly labelled or was given no label at all in 90% of solution attempts (Step 5), is some evidence of student lack of reliance on diagram analysis for processing the vector solution.

In Step 17, incorrect statements for the value of vertical tension component T_Y was observed in 52% of solution scripts. In this step, 21% of scripts did not record a value for the T_Y vector. It is noted here, that a correct calculation of this component vector is an essential prerequisite to finding the resultant chain tension T_R. Referring to Step 6, it is noted that the same frequency, 21% omitted or skipped steps, occurred by student failure to resolve the components of T_R diagrammatically. This may help correlate the absence of that diagram for T_Y, with the failure to record the necessary numerical value for T_Y, in Step 17.

From the comparative analysis of graphs for Levels 1 and 2 (Figure 22), two differences were noted in the first half of the drawbridge solution part (i). They dealt with conventional assumptions for: (1) location of centre of gravity in Step 10, and (2) their application to problem solving using the drawbridge mass. For example, when substituting numerical values into the equation of moments in Step 15 certain assumptions were made.

Though it is not stated in the wording of the problem, an expert problem solver would assume the drawbridge to be of uniform mass. Application from this assumption would locate the centre of gravity half way along the drawbridge. There is no physical or engineering reason why any drawbridge should have uniform mass. So these conventional assumptions have to be explicitly taught to novice problem solvers. (Larkin, 1981; Taylor, 1982.)

In Level 2 solution scripts, 28% of students (Table 9 below) failed to position correctly, or did not draw the centre of gravity vector in their drawbridge diagram (Step 8). The assumption of that vector is essential to taking moments about the hinge pivot, and for eventual calculation of the vertical component of the drawbridge tension T_y.

A detailed error frequency record for handling this basic principle of mechanics is presented next, prior to discussion of comparative errors between the two levels.

Table 9

Record of incorrect statements of errors and omissions, related to the conventional assumption that a beam has uniform mass (in this case, the drawbridge beam)

Step Number	Step Description	Level 1 Errors Code 0,1	Level 1 Errors Code 2	Level 2 Errors Code 0,1	Level 2 Errors Code 2
Sample size:		65	65	103	103
	(Centre of gravity)				
8	cg. vector	0	0	4	12
9	cg. value	0	2	1	7
10	cg. location	1	0	16	13
Total Error Occurrence ..		1	2	21	32

From Table 9, there may be seen a difference between the number of student solution script errors for Level 1 compared with those of Level 2. Solutions from Level 2 had many times the number of omissions and errors in locating the centre of gravity for the drawbridge. Only one omission error of location and two incorrect values occurred in all of Level 1 solution steps for centre of gravity.

Discussion of these steps leads into the whole question of making reasonable assumptions, recognising implied information of a problem statement. This comes under the heading of problem solving interpretation (Reif, 1982). Level 1 students had very little trouble in arriving at a correct value of 20,000N for the weight of the drawbridge. In this group, there was not one incorrect statement of location at 30 metres from the drawbridge pivot.

For Level 2 students, finding the correct location for centre of gravity of the drawbridge was a common difficulty. From Table 9 above, 16% of solutions omitted a reference diagram and 13% of solutions located its position incorrectly. The difference in error frequency is so marked in these early steps, as to suggest that it could become a distinguishing characteristic for "very good" problem solvers, because not one incorrect assumption was made by any Level 1 student in this regard, whereas many incorrect statements and omissions occurred in Level 2 solutions. Further discussion of this finding occurs in the conclusion.

Comparative analysis of Level 1 with Level 2 was facilitated by plotting the adjusted tabulations of each level on the same column graph in Figures 22 and 23. This presented a progressive comparative analytic display for the percentage of steps completed correctly. Some characteristic differences between the two levels were distinguishable as a result of this graphical comparison. Detailed discussion of this comparative analysis and the separating characteristics of the two levels will be presented in tables and discussion further on.

Level 1 contained 65 solution attempts. Level 2 contained 103 solution attempts.

The early steps (2 and 3) contained only two incorrect statements. On this account, these steps did not show up as an obstacle to student progress. However, when the number of skipped steps is

considered, a different result is found. Steps 2 and 3 are diagram analysis steps, where 12% of students from level 1 and 12% from Level 2 omitted to label the drawbridge length, while 11% from Level 1 and 18% of Level 2 students omitted to label the distance from pivot, to chain contact with the bridge. These errors of omission occurred, despite the fact that their values were recorded as part of the problem statement. The conclusion drawn from these results is that the lack of analytic detail in diagrams seems to be related to subsequent obstacles to progress.

Steps 8, 9 and 10 have been discussed earlier in this Chapter. They involved errors of vector location and the value for the drawbridge centre of gravity. These steps were a source of incorrect statement in diagram analysis for 20% of students in Level 2. Almost no errors, in these steps, were made by Level 1 students.

A substantial source of error for Level 2 students in diagram analysis, occurred at Step 14. This was related to the taking of moments about the pivot and stating the related equation of moments. Incorrect statements were recorded by 30% of students from Level 2.

The author had initially decided that a correct answer should arise from a logical explanation, originating with rule statements and then followed by correct substitution and mathematical simplification. This would have provided some evidence for each student's understanding, in the process of physics problem solving.

It would have been accessible evidence of a student's ability to communicate a logical argument of physics.

Fourteen errors of diagram analysis for Level 1 students, were recorded in only two of the steps, as summarised in Table 10 below. Each of the errors in this Table led to a subsequent miscalculation involving an incorrect final answer.

Table 10

Error frequency record for Level 1 and Level 2 Student attempts on solution steps involving omission and incorrect statement of diagrams

(Record of Codes 0, 1, 2)

Step Number	Step Description	Number of Errors Level 1	Level 2
3	T_R diagram	7	20
6	T_Y diagram	7	29
Sample sizes		65	103

As may be seen in Table 9, the basic assumption requiring location of the centre of gravity on the drawbridge, was very well understood by Level 1 students. Only two out of the 65 scripts recorded a misplacement. This result is in contrast with the seven incorrect statements in the scripts of Level 2.

The difference noted for Level 2 solution attempts, is that almost twice as many Level 2 students failed to position correctly, or failed to recognise the need for, a centre of gravity vector, to account for the drawbridge weight as supplied in the problem statement. This error frequency suggests the inability to conceptualise such a force in this particular case. It involves the concept of a basic principle of mechanics.

From Table 10, can be seen the difference in diagram analysis between Level 1 and Level 2. Steps 3 and 6 involve the resolution of a resultant force into its vertical and horizontal components. Allowing for sample size difference, Level 2 solution scripts had twice as many errors as Level 1, in the process of drawing and labelling component vectors.

SUMMARY OF RESULTS

The TAE Physics syllabus claims to be attempting to assess understanding of physics principles and their applications. It was therefore decided to regard omitted steps as errors of omission or as substantial errors in problem solving communication. These errors may also be regarded as an incomplete presentation in the logical sequence of solution evidence.

For Level 1 and Level 2 solution attempts, the most obvious occurrences were errors of omission. The steps omitted could be described as explanatory steps (as opposed to calculation steps). It involved the failure to state or indicate the rule or principle being applied, either by way of a formula or in words. It was frequently left to the research marker to guess what a particular equation represented by way of explanatory solution.

An important result of this study is that many of the same kinds of basic conceptual difficulties with diagram analysis of vectors are exhibited by all levels of student problem solving ability. Serious problems concerning diagram analytic understanding, have been presented by student problem solvers, whose TAE marks indicated that they have high ability.

Relating the graphical peaks of correct statements in Figures 22 and 23, to the types of errors involved, has helped to provide a picture of the sources of student difficulty in tackling the drawbridge solution. Incorrect statements, recorded against solution steps of diagram analysis, are more numerous for both Levels 1 and 2, than for any other type of solution step.

The graphical record of Figures 22 and 23 emphasised the points of common difficulty. These occurred in not drawing a correctly connected and labelled diagram. Subsequent related errors appeared during final substitution and simplification in equations. For Level 1 students, even the very best problem solvers (who seemed to understand the problem) failed often in their visual translation

from the written information of the problem statement. From a total of 46 errors made by Level 1 students, 37% of those errors occurred within the region of visual translation to a labelled diagram.

Solution steps were not accompanied by adequately detailed computations. Students generally showed little respect for explanatory detail. This finding is evidenced by the high frequency of no response to solution steps and deliberately skipped steps.

RESULTS OF THE JIBCRANE SOLUTION ANALYSIS

Introduction

In the process of scoring student solutions, four distinctly different methods of solution were separable and are named below.

Method 1 Vector component analysis

Method 2 Moment equilibrium analysis

Method 3 Vector triangle calculation

Method 4 Scaled triangle construction.

Each method was sufficiently different in approach, and in its number of steps, to require solutions to be analysed separately according to method.

In all methods, the scoring procedure took into account that physics principles or rules had to be recognised by students and applied in sequence. A clear statement of these rules was regarded as necessary, before introducing related equations. Clearly labelled diagrams were considered a prerequisite to adequate explanation and logical reasoning. These points are central to the model solution sequences, presented in Chapter 3.

Methods 1 and 2 of the jibcrane solution emphasised the aspects of vector analysis and moment equilibrium. Method 3 is a mathematical solution, relying on trigometric properties of the sine rules applied to a vector triangle. Method 4 solution relies on the construction of an accurately scaled drawing of the vector triangle.

This thesis is about physics problem solving. For that reason, solution scripts of Method 1 and Method 2 have been fully analysed. The mathematical solutions of Method 3 and Method 4 have not been analysed and so, have been omitted in discussion.

JIBCRANE SOLUTION

A detailed discussion on choice of method will not be entered into as part of this thesis. Table 11 presents the student choice of method in terms of different solution methods that were identified for the problem.

Table 11

Choice of method and number of students for the jibcrane solution

Method	Number of Students			
	Total	Level 1	Level 2	Level 3
1. Vector Component Analysis	243	42	71	130
2. Moment Equilibrium Analysis	85	29	39	17
3. Vector Triangle Calculation	1659	292	321	1046
4. Scaled Triangle Construction	30	30	0	0

The need to analyse separated levels of solution attempts, according to performance ability, was discussed in the previous section and has been applied to the analyses of the jibcrane solution.

RESULTS OF JIBCRANE SOLUTION ANALYSIS, METHOD 1

Level separation was commenced by making a table of error frequencies, tallied from each student performance. Table 12 summarises that frequency distribution and Figure 26 presents the related histogram.

Table 12

Summation of errors in the jibcrane solution scripts for Method 1 and the number of students with that error frequency

Error Total	Student Number	Error Total	Student Number
0	0	16	15
1	0	17	18
2	0	18	19
3	0	19	12
4	0	20	11
5	0	21	14
6	5	22	19
7	4	23	17
8	6	24	19
9	3	25	12
10	6	26	8
11	8	27	6
12	5	28	6
13	5	29	6
14	10	30	0
15	9		

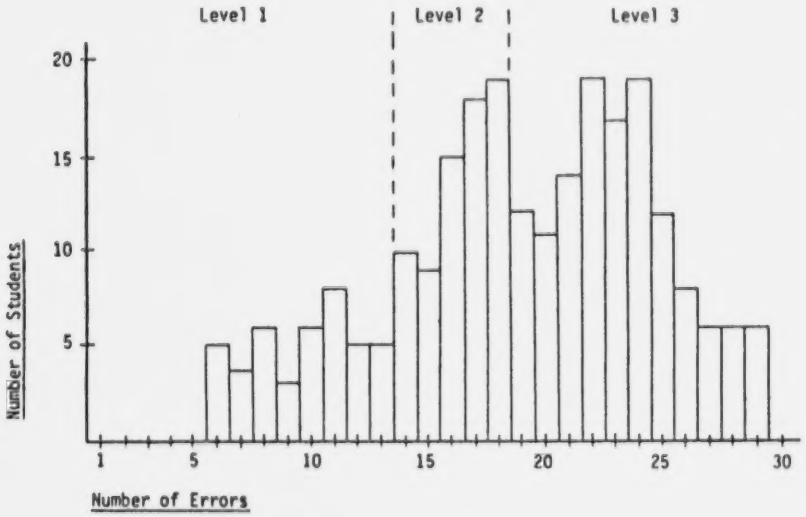


Figure 26

Summation of Errors in the Jibcrane Solution Scripts for Method 1 and the Number of Students with that Error Frequency.

From the histogram in Figure 26, three levels of solution performance were separated. This arbitrary separation followed the same procedure used for separating levels in the drawbridge solution scripts. The reason for this separation of levels was to cull out those particularly unsuccessful solutions from further detailed analysis. Decisions on the arbitrary subdivisions of ability were, as before, based upon a subjective evaluation made by the author. The boundaries of each level were not regarded as critical. They were made to provide a rough separation between the "correct", the "partially correct" and the "unsuccessful" solution attempts.

A frequency of zero to thirteen errors was decided upon, to isolate Level 1. From fourteen to eighteen errors of omission, skipped step or erroneous statement, was the frequency used to isolate Level 2. From 243 students who attempted the jibcrane solution by Method 1, 42 scripts were selected for Level 1, 71 solution scripts were placed in Level 2 and the remaining 130 scripts were placed in Level 3. This last level was removed from further analysis. The error coding translation is the same as for the drawbridge error analysis.

Analysis of errors in the jibcrane solution scripts, Method 1

General analysis of Method 1 solution scripts, based on error code summary for each solution step, is presented in Table 13. The error code summary for Level 1 is followed by a percentage analysis. Steps coded 1 and 3 have been combined to record the percentage of steps completed correctly. Steps coded 0 and 2 have been combined to record the percentage of steps completed incorrectly. The results are followed by a composite table, which combined the results of Level 1 and Level 2.

Table 13

Summary of Errors in the Jibcrane Solution Scripts
Vector Component Analysis. Solution Method 1

	LEVEL 1 : N = 42								LEVEL 2 : N = 71								COMPOSITE : N = 113							
Step	Error Code 0	1	2	3	0&2	%	1&3	%	Error Code 0	1	2	3	0&2	%	1&3	%	Error Code 0	1	2	3	0&2	%	1&3	%
1	0	3	0	39	0	0·0	42	100	1	3	0	67	1	1·4	70	98·6	1	6	0	106	1	0·9	112	99·1
2	0	0	0	42	0	0·0	42	100	2	1	0	68	2	2·8	69	97·2	2	1	0	110	2	1·8	111	98·2
3	0	0	0	42	0	0·0	42	100	2	1	0	68	2	2·8	69	97·2	2	1	0	110	2	1·8	111	98·2
4	1	19	0	22	1	2·4	41	97·6	4	27	1	39	5	7·0	66	93·0	5	46	1	61	6	5·3	107	94·7
5	0	38	0	4	0	0·0	42	100	2	59	0	10	2	2·8	69	97·2	2	97	0	14	2	1·8	111	98·2
6	0	0	0	42	0	0·0	42	100	2	4	5	60	7	9·8	64	90·2	2	4	5	102	7	6·2	106	93·8
7	1	18	0	23	1	2·4	41	97·6	4	28	2	37	6	8·4	65	91·6	5	46	2	60	7	6·2	106	93·8
8	0	39	0	3	0	0·0	42	100	6	57	3	5	9	12·7	62	87·3	6	96	3	8	9	8	104	92
9	0	0	0	42	0	0·0	42	100	6	4	4	57	10	14·1	61	85·9	6	4	4	99	10	8·8	103	91·2
10	1	18	0	23	1	2·4	41	97·6	4	29	5	33	9	12·7	62	87·3	5	47	5	56	10	8·8	103	91·2
11	0	38	0	4	0	0·0	42	100	5	58	4	4	9	12·7	62	87·3	5	96	4	8	9	8	104	92
12	0	0	0	42	0	0·0	42	100	5	4	13	49	18	25·4	53	74·6	5	4	13	91	18	15·9	95	84·1
13	0	5	0	37	0	0·0	42	100	2	6	0	63	2	2·8	69	97·2	2	11	0	100	2	1·8	111	98·2
14	2	25	0	15	2	4·8	40	95·2	4	47	0	20	4	5·6	67	94·4	6	72	0	35	6	5·3	107	94·7
15	0	0	1	41	1	2·4	41	97·6	2	0	2	67	4	5·6	67	94·4	2	0	3	108	5	4·4	108	95·6
16	0	19	0	23	0	0·0	42	100	5	29	0	37	5	7·0	66	93·0	5	48	0	60	5	4·4	108	95·6
17	0	39	1	2	1	2·4	41	97·6	3	55	0	13	3	4·2	68	95·8	3	94	1	15	4	3·5	109	96·5
18	0	0	1	41	1	2·4	41	97·6	1	2	5	63	6	8·4	65	91·6	1	2	6	104	7	6·2	106	93·8
19	0	23	0	19	0	0·0	42	100	13	31	0	27	13	18·3	58	81·7	13	54	0	46	13	11·5	100	88·5
20	0	3	1	38	1	2·4	41	97·6	19	6	3	43	22	31·0	49	69·0	19	9	4	81	23	20·4	90	79·6
21	0	0	0	42	0	0·0	42	100	18	2	10	41	28	39·4	43	60·6	18	2	10	83	28	24·8	85	75·2
22	1	3	1	47	2	4·8	40	95·2	21	4	15	31	36	50·7	35	49·3	22	7	16	68	38	33·6	75	66·4
23	0	3	1	38	1	2·4	41	97·6	17	3	27	24	44	62·0	27	38·0	17	6	28	62	45	39·8	68	60·2
24	0	0	3	39	3	7·1	39	92·9	16	1	38	16	54	71·8	17	28·2	16	1	41	55	57	50·4	56	49·6
25	0	1	6	35	6	14·3	36	85·7	23	3	37	8	60	84·5	11	15·5	23	4	43	43	66	58·4	47	41·6
26	0	3	12	27	12	28·6	30	71·4	21	1	42	7	63	88·7	8	11·3	21	4	54	34	75	66·4	38	33·6
27	1	30	6	5	7	16·7	35	83·3	21	29	17	4	38	53·5	33	46·5	22	59	23	9	45	39·8	68	60·2
28	1	0	7	34	8	19·0	34	81·0	22	2	43	4	65	91·5	6	8·5	23	2	50	38	73	64·6	40	35·4
29	1	0	7	34	8	19·0	34	81·0	22	2	45	2	67	94·4	4	5·6	23	2	52	36	75	66·4	38	33·6
30	1	3	10	28	11	26·2	31	73·8	22	3	42	4	64	90·1	7	9·9	23	6	52	32	75	66·4	38	33·6

From the error code summary in Table 13, column graphs were constructed in Figure 27. These graphical analyses were recorded on the same graph paper, to permit a comparison and contrast between problem solving performances of Level 1 and Level 2. The graphs show where major common difficulties have occurred.

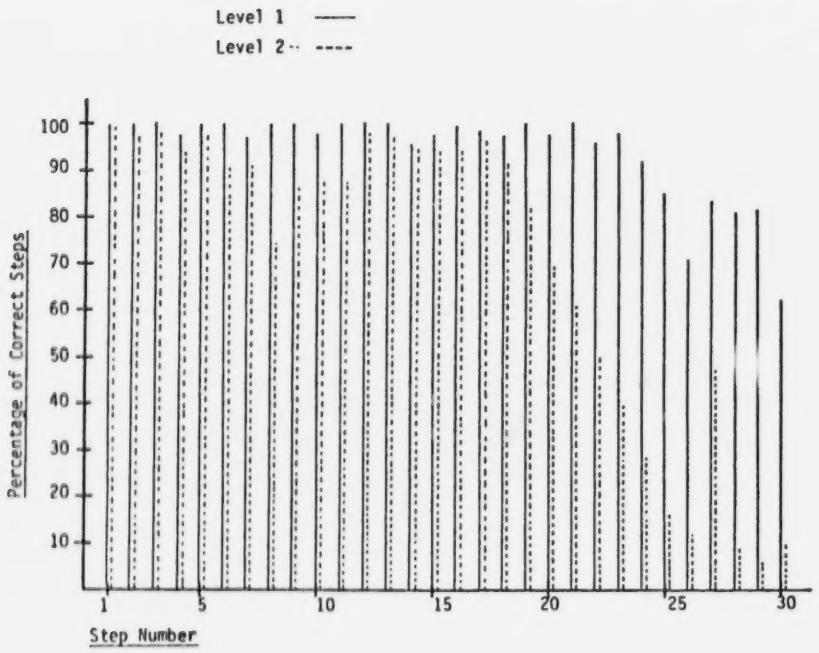


Figure 27

Jibcrane Solution Method 1

Comparative Column Graphs for Levels 1 and 2
Percentage of Steps Completed Correctly

After recording the summary of errors, it was important to draw out of Table 13, those errors (code 2) where definitely incorrect statements had been recorded. From Table 13, Figures 28, 29 and 30 (following), were constructed, to illustrate code 2 errors for composite, Level 1 and Level 2 results.

The purpose of these graphs was to identify the common steps of erroneous statement. They also permit comparison and contrast between Level 1 and Level 2 results.

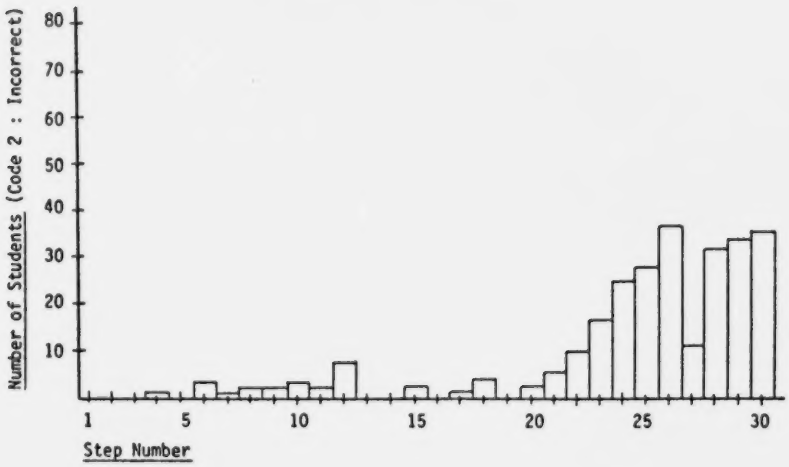


Figure 28

Summation of Code 2 Errors for the Jibcrane Solution Scripts.
Composite of Levels 1 and 2 for Method 1.
Code 2 indicates step stated incorrectly.

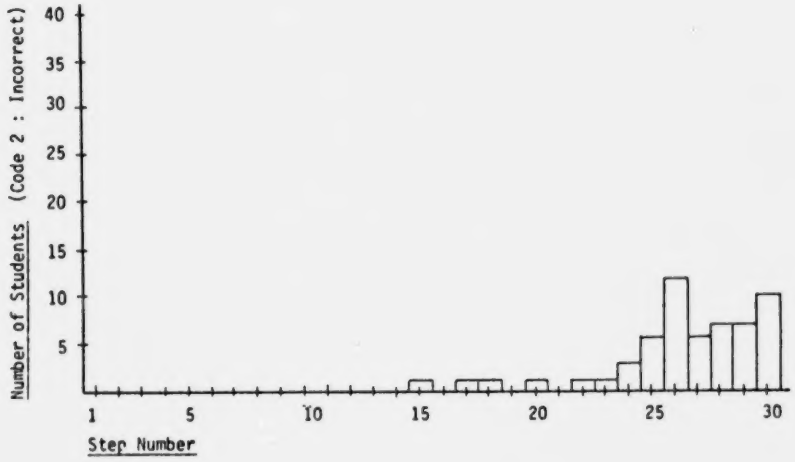


Figure 29

Summation of Code 2 Errors for the Jibcrane Solution Scripts.
Student Level 1 for Method 1.
Code 2 indicates step stated incorrectly.

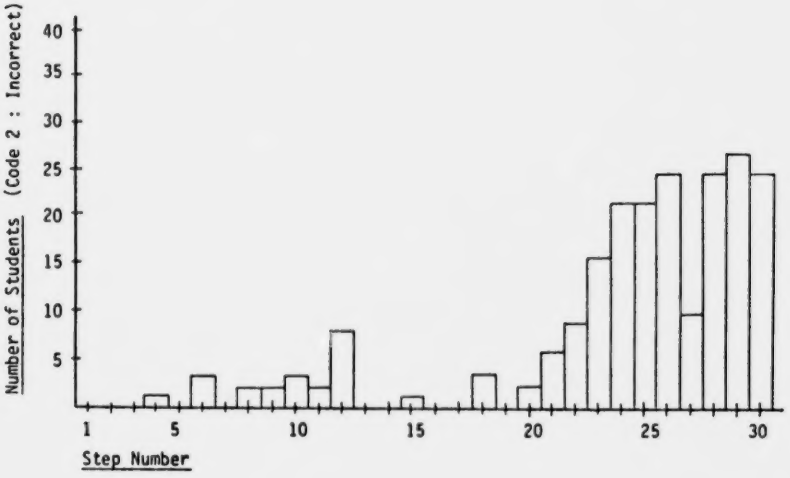


Figure 30

Summation of Code 2 Errors for the Jibcrane Solution Scripts.
Student Level 2 for Method 1.
Code 2 indicates step stated incorrectly.

Discussion of Results of Error Analysis in

Jibcrane Solution, Method 1

From the population of 2017 who attempted the jibcrane solution, 243 students used the method of vector component analysis. Of these, 42 solution scripts were selected for Level 1 and the next 71 solution scripts were put into Level 2. The remaining 130 scripts, called Level 3, were rejected from further analysis.

By reference to the graph in Figure 28, for composite summation of Code 2 errors, it may be seen that erroneous statements were grouped in two stages. At Step 6, there began to appear simple errors involving the angle properties of triangles and simple trigonometric relationships of a right angled triangle.

$$\text{viz.} \quad \sin 30° = \frac{Xj}{F_j}$$

The next major obstacle occurred after Step 19, the statement of the law of equilibrium for horizontal components:

$$\Sigma F_H = 0.$$

Steps 22 and 23 required substitution of symbols and values into equations of equilibrium, as an application of the general principle of moments for horizontal components (Step 22) and for vertical components (Step 23). Each of these steps was a major obstacle to solution progress.

From the composite results of Table 13, it may be seen that no response or an incorrect statement was recorded in 34% of cases for Step 22, and in 40% of cases for Step 23. Each of these steps depended for its correctness upon an accurate diagram analysis of the jibcrane component vectors. In many cases, there was no diagram analysis and no related equation of equilibrium.

Substitutions required for jib force and tie force in Steps 24, 25 and 26 relied upon a correct analysis into vertical and horizontal components: (see diagram in Chapter 3).

$$F_t.\cos 75° + 10^4 = F_j.\cos 30° \quad \text{(Step 24)}$$

from Step 22: $F_j = F_t\ (1{\cdot}9318)$ (Step 25)

therefore, $F_t = 7{\cdot}072 \times 10^3 N$ (Step 26).

The consequential substitution of Step 22 into Step 24, in order to arrive at Step 26, was incorrectly handled by 66% of Levels 1 and 2 students, as can be seen from the composite column in Table 13 and from the graph of Code 2 composite (Figure 28).

These findings will be analysed further in the handling of Level 1 separately from Level 2. When each set of separated levels was analysed, trends in error occurrence and items of interest became evident.

Level 1 contained 42 solution scripts. Erroneous statements were concentrated in the areas which required a statement of equilibrium between vertical component vectors. The first major obstacle to Level 1 solution progress occurred at Step 24, with the incorrect labelling of component vectors on the jibcrane diagram. These errors related closely to the subsequent substitution of incorrect values into trigonometric equations. This led to incorrect and unrealistic solutions of the required forces on the jib and on the tie (Steps 25 to 30). In the case of unrealistic answers for each force, there was no written evidence in any Level 1 paper to suggest that students had recognised the existence of any error in magnitude.

More than half of Level 1 solution errors were directly associated with the incorrect analysis and labelling of horizontal and vertical component vectors. Major factors in error occurrence appeared to be the absence of effectively drawn analytical or freebody diagrams and the misuse of trigonometric properties of triangles associated with diagram analysis of the vectors involved.

Level 2 for Method 1 contained 71 solution scripts (Table 13). Students in Level 2 began to experience difficulties quite early in the solution process. Step 5 required recognition of a 30° angle between the jib and the load. Only 10 students out of 71 inserted this value into their diagrams. An incorrect statement of angle size was made in Step 6 by 7% of Level 2 students. In Step 8, 13% of Level 2 students arrived at an incorrect trigonometric equation, when they failed to recognise an angle of 15° between the tie force and its horizontal component.

Finding the trigonometric relationship, in Step 12, for the vertical component of the tie force Y_t, was skipped or handled incorrectly by 31% of Level 2 candidates. As in Level 1, analysis of vectors into vertical and horizontal vector components was also a major obstacle to Level 2 solution progress, for the jibcrane problem solvers in Method 1.

Of equivalent significance in Level 2 solutions, was the 13% error occurrence in establishing the value of 75° for the angle between the tie and the supporting wall (Step 11). By comparison, there were no errors made at Step 11 in Level 1 scripts. It will be noted that all of the errors discussed so far are directly associated with diagram analysis.

Most of the values required elementary insight to recognise, label and apply the angle properties of a right angled triangle. To see ahead of such properties and to recognise their usefulness as part of the global approach to solving this problem, is a quality assigned to an experienced or expert problem solver (Larkin, 1979).

> "The second difference between experts and novices concerns how physical principles are stored in memory. The data discussed below suggest that in an expert's memory, principles are not stored individually, but a group of principles are connected and stored as a "chunk". For example, various force laws and kinematic principles seem to be closely chunked with Newton's equation of motion (F = ma), so that whenever F = ma is applied, it is almost automatic to consider also applying the other principles in this chunk.
>
> In contrast, the novice I have studied in detail apparently does not have such tightly connected chunks. After applying F = ma, he must search through a large group of principles to decide what to do next."
>
> (Larkin, J.H., 1979, p. 286.)

The above quotation provides some background to help distinguish between Level 1 and Level 2 styles of error. Level 2 students seemed to be making errors every few steps. Perhaps this occurrence is related to their possible inability to see ahead, and recall physical principles as blocks of related information and strategies. Level 1 solution scripts have generally seemed to recognise the need for triangle properties and values, as well as their trigonometric relationships. These more competent students gave evidence of a certain degree of foresight, which Larkin would call the level of block recall of principles. These "chunk" recalls were illustrated diagrammatically and were stated in equations. Errors at Level 1 occurred later on, with the consequential application of these trigonometric properties in vector equations.

This of course, is at the level where experts might also be expected to have some difficulty in applying consequential relationships correctly in equations such as Step 24:

$$F_t.\cos 75° + 10^4 = F_j.\cos 30°.$$

An unexpected source of error for Level 1 solution scripts occurred in Steps 25 and 26 where 29% of students made errors of simple algebraic crossmultiplication and simplification. This comparatively large error occurrence is in an area which most physics teachers would have regarded as assumed knowledge or entry skills to a basic physics course. In fact, more errors occurred in crossmultiplication and simplification (from Steps 25 to 30) than for the entire previous 24 steps leading up to the solution. And this, for Level 1 student problem solvers was quite unexpected. It is also noted that Level 2 students, in Steps 25 to 30, made approximately three times the number of error statements as did Level 1 problem solvers. No clear explanation was forthcoming as to the reason for such unrealistic answers.

Time constraints are one possibility leading to careless algebraic manoeuvres. These findings are consistent with the statistical data from the drawbridge sample analysis.

> "Though at times it seems that students have no appreciation for the distinction between "wrong" and "ridiculous", the actual difficulty is that few students anticipate solutions or evaluate solutions in terms of these expectations. The cause of this is a significant lack of knowledge of perspectives. As a store of this type of information builds, anticipation and evaluation becomes spontaneous."
>
> (Brown, J.M., 1980, p. 115.)

The above quotation may provide some degree of insight to the characteristic distinction in production of unrealistic answers between Level 1 and Level 2 student performances.

A graphical comparative analysis of step number and error occurrence has been illustrated for Method 1, to present some ideas about general trends in error frequency. Comparative graphing may also assist in identifying common obstacles, step by step. This may help to answer some of the questions of this thesis, especially as to types of difficulties common to all student solutions.

Level 1 students skipped many initial solution steps, as may be seen by referring to Code 1 in Table 13. It is important to note however, that these omissions seemed to have no effect on the arrival at a correct final answer for Level 1 students. Level 2 students, in contrast, were often not able to recover from their early errors of omission. Interpretation of this characteristic difference suggests that Level 1 students may have retained the omitted steps of diagram analysis in their global concept of the solution process and could refer to it mentally for later use.

Level 2 students often did not draw correct information at a later step, from previously omitted diagram analysis statements. This suggests a lack of continuity between pieces of information or the absence of a global concept for effective solution strategy (Larkin, 1979).

In summary then, the two main difficulties for the best student problem solvers (Level 1) seemed to lie in the drawing of relevant closed triangles of equilibrium by orthogonal vector component analysis and in the correct calculation of component vectors using the trigonometric properties of triangles. Consequential equations of equilibrium for the vertical component vectors, were directly related to errors and omissions in diagram analysis.

For Level 2 students, major difficulties appeared within the first few steps of the solution process. Errors involved elementary properties of triangles and vector diagrams.

Thereafter, Level 2 solution scripts showed evidence of encountering the same obstacles as Level 1. They showed a threefold increase in error occurrence over Level 1, in correctly illustrating the relevant closed triangles for vector component analysis. A similar threefold increase in error occurrence for Level 2 over Level 1 was tabled for correct calculation of component vectors using trigonometric properties of triangles.

RESULTS OF JIBCRANE SOLUTION ANALYSIS, METHOD 2

Level separation was commenced with a table of error frequencies, tallied for each student performance. For Method 2, called the moment equilibrium method, the summary frequency distribution is presented in Table 14 and its related histograph is represented in Figure 31.

Table 14
Summation of errors in the jibcrane solution scripts for Method 2 and the number of students with that error frequency

Error Total	Student Number	Error Total	Student Number
0	5	5	12
1	1	6	16
2	5	7	11
3	7	8	15
4	11	9	12

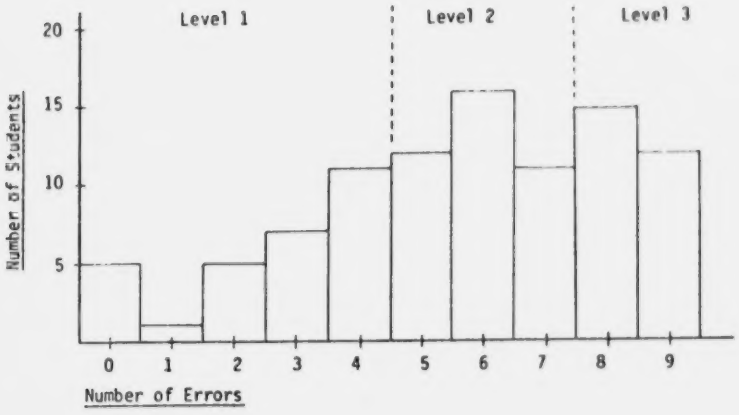


Figure 31
Summation of Errors in the Jibcrane Solution Scripts for Method 2 and the Number of Students with that Error Frequency.

From the histogram in Figure 31, three levels of solution performance were separated. This arbitrary separation followed the same procedure used for separating levels of performance in Method 1 of jibrane solution scripts.

Again, reasons for this separation was to cull out those particularly unsuccessful solutions from further detailed analysis. The other reason was to establish a rough separation between the 'successful' and the 'partially successful' student solution scripts.

Method 2 has only nine steps. A decision was made that zero to four errors should represent Level 1, five to seven errors should represent Level 2 and eight or more errors should represent Level 3.

As was presented for Method 1, a similar error code summary is presented in Table 15 for Method 2 of the jibcrane solution. The Table includes an error code summary of Level 1, then Level 2 and finally, a composite analysis of Levels 1 and 2 combined. The error code translation for this Table is the same as for Method 1 and has been fully detailed in Chapter 3.

In Table 15, the error code summary is followed by a percentage analysis. Steps coded 1 and 3 have been combined to record the percentage of steps completed correctly. Steps coded 0 and 2 have been combined to record the percentage of steps completed incorrectly. These results are followed by a composite Table which combines the results of Levels 1 and 2.

Table 15

Summary of Errors in the Jibcrane Solution Scripts
Moment Equilibrium Analysis. Solution Method 2

Step	LEVEL 1 : N = 29								LEVEL 2 : N = 39								COMPOSITE : N = 68							
	Error Code				0&2	%	1&3	%	Error Code				0&2	%	1&3	%	Error Code				0&2	%	1&3	%
	0	1	2	3					0	1	2	3					0	1	2	3				
1	0	0	0	29	0	0	29	100	3	2	1	33	4	10·2	35	89·8	3	2	1	62	4	5·9	64	94·1
2	0	3	0	26	0	0	29	100	6	17	0	16	6	15·4	33	84·6	6	20	0	42	6	8·8	62	91·2
3	0	6	0	23	0	0	29	100	1	11	0	27	1	2·6	38	97·4	1	17	0	50	1	1·5	67	98·5
4	0	0	7	22	7	24	22	76	5	0	19	15	24	61·5	15	38·5	5	0	26	37	31	45·6	37	54·4
5	0	0	9	20	9	31	20	69	8	1	21	9	29	74·4	10	25·6	8	1	30	29	38	55·9	30	44·1
6	0	11	0	18	0	0	29	100	20	14	0	5	20	51·3	19	48·7	20	25	0	23	20	29·4	48	70·6
7	0	9	0	20	0	0	29	100	18	11	2	8	20	51·3	19	48·7	18	20	2	28	20	29·4	48	70·6
8	1	0	13	15	14	48·2	15	51·8	19	0	17	3	36	92·3	3	7·7	20	0	30	18	54	79·4	14	20·6
9	1	0	15	13	16	55	13	45	20	0	18	1	38	97·4	1	2·6	21	0	33	14	54	79·4	14	20·6

From the error code summary in Table 15, column graphs were constructed in Figure 32. These graphical analyses were recorded on the same graph page to permit a comparison and contrast between problem solving performances of Level 1 and Level 2. The graphs show where major common difficulties have occurred.

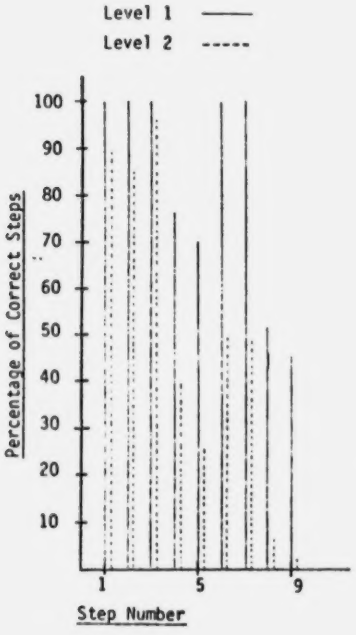


Figure 32
Jibcrane Solution Method 2
Comparative Column Graphs for Levels 1 and 2
Percentage of Steps Completed Correctly

Analysis of errors in the jibcrane solution, Method 2

General analysis of Method 2 solution scripts commenced with an error code summary (Table 15) for each solution step. Column graphs in Figure 32 recorded the percentage of steps completed correctly for Method 2.

The graphs of Code 2, for incorrect statements (Figures 33, 34 and 35 following) will be the focus of this analysis. As before, the presentation of these graphs still leaves open for inspection, any other errors not dealt with in detailed discussion.

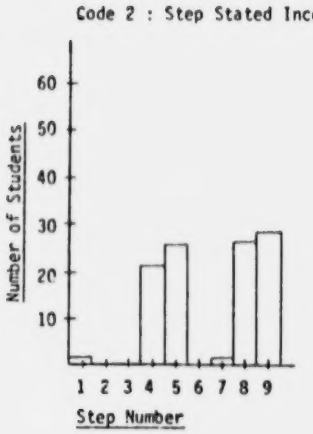


Figure 33

Summation of Code 2 Errors in the Jibcrane Solution Scripts.

Composite of Levels 1 and 2 for Method 2.

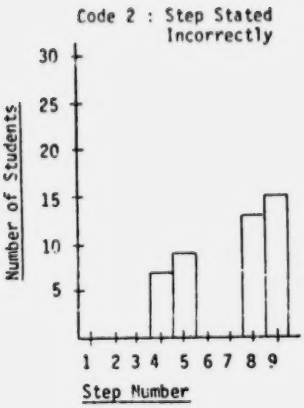


Figure 34

Summation of Code 2 Errors in the Jibcrane Solution Scripts

Student Level 1 for Method 2.

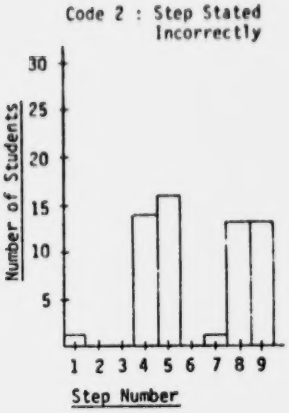


Figure 35

Summation of Code 2 Errors in the Jibcrane Solution Scripts.

Student Level 2 for Method 2.

Jibcrane Solution : Method 2 Background Discussion

At this stage, it is appropriate to insert a short background discussion to support the interpretation (in Chapter 5) of results to be discussed next in Method 2. It will especially illuminate some suggestions made while comparing Level 1 (experienced problem solvers) with Level 2 (less competent performers).

There are some characteristic tendencies which can help distinguish between the expert problem solver and a comparative novice who has a lesser level of problem solving experience. An expert problem solver, according to Larkin (1981), proceeds from reading the problem statement to a translational sketch of the physical situation described. Next follows a diagram to reflect the entities of physics (such as forces and moments) to be used in solving that problem. (In the jibcrane problem, this takes the form of a freebody diagram.) From the freebody diagram, arises a set of equations which are then used to solve the problem with substituted values.

A novice problem solver, continued Larkin (1981), commonly tries to construct equations directly from the statement of the problem and even more commonly skips both the physical situation diagram and the freebody diagram. The experienced problem solver uses the physical diagram and the freebody translation to test the appropriateness of a particular method before proceeding further.

The skilled problem solver separates the task of identifying components from the task of relating them quantitatively in equations. In the jibcrane solution this might permit the experienced problem solver to decide whether to handle the solution by orthogonal vector analysis or by moment equilibrium analysis.

Discussion of Results of Error Analysis in Jibcrane Solution, Method 2

From the population of 2017 who attempted the jibcrane solution, 85 students chose the method of moment equilibrium analysis. From this population of 85 scripts, 29 solution scripts were selected for Level 1 and the next 39 solution scripts were placed in Level 2. The remaining 17 scripts in Level 3 were rejected from further analysis.

In Figure 33, the graph for composite summation of errors coded 2, it can be seen that incorrect statements were concentrated at Steps 4, 5 and Steps 8, 9. Each of these pairs of related steps required an equation for moment equilibrium, followed by substitution of algebraic values and simplification.

The general appearance of the composite code 2 graph (Figure 33) suggests that out of the nine recorded steps, there was a major obstacle at Step 4 and Step 8 respectively.

The difficulty in Step 4 may be related more closely to the 29% omission errors (Table 15) for diagram analysis in Step 2 (Chapter 3). It may also have originated with the 25% omission errors in the general statement of equilibrium about any pivot in Step 3.

Step 4 required a statement for the equation of equilibrium about the pivot:

$$F_t.\cos 45°.PQ = F_m.\cos 60°.PQ \quad \text{(Step 4, Figure 18)}$$

Out of 68 students from Levels 1 and 2, the above equation for moment equilibrium (Step 4) was stated incorrectly in 26 solution scripts. This represents a 38% occurrence of incorrect statements at Step 4.

The number of errors for Step 4 in the equation of equilibrium, was more closely investigated when the separate analysis of Levels 1 and 2 were compared. In Level 1, 10% and 20% of students skipped steps for Steps 2 and 3 respectively. This was followed by 24% incorrect statements (Code 2) in Level 1, for the crucial equation statement of Step 4. In Level 2, errors of omission and skipped steps in Steps 2 and 3 had risen to 59% and 31% respectively. This was followed by 49% incorrect statements, for the key equation of moments in Step 4 (Figure 18).

In Step 8, a second key equation of moments was required, about T, the point of contact between the tie and the supporting wall. The composite graph Code 2 (Figure 33) recorded incorrect statements in 44% of student attempts on Step 8. This equation should have arisen from information recorded in a labelled diagram analysis of related moments. Reverting to Step 6 for that diagram analysis, the composite error frequency in Table 15, recorded 66% as no response or skipped step, for the relevant labelled diagram.

The following step (Step 7) required a statement of equilibrium for moments about T. Omission errors and skipped steps were recorded at 56% for Step 7.

Level 1 contained 29 solution scripts. Incorrect statements were concentrated in Steps 4 and 8, where an equation of moment equilibrium was required in each case. Step 5 and Step 9 required consequential substitution of values generated from the freebody diagram of Step 2.

It can be seen from Figure 34 that there were no incorrect statements recorded in Step 2 for the diagrams of Level 1 solution scripts and there were only 10% of solutions in Level 1 for which a diagram had not been drawn. Step 3 omission errors of 21% (Table 15) involved the failure to record a moment equilibrium statement. The errors of these two steps may have had an influence on the major errors at Step 4 and Step 8. The related errors seem to have originated by way of incorrect trigonometric relationships for diagrammatic resolution of the tie and weight force components.

Level 2 contained 39 solution scripts. Level 1 had very few errors of omission. In Level 2, by contrast, there was a very large increase in the occurrence of omissions. For Step 2, Level 1 had recorded 10% omission errors in the diagram. For Level 2, errors of omission had risen to 59% in drawing and labelling a freebody diagram.

Such a large difference in this type of error occurrence needs special attention. The characteristic differences between experienced physics problem solvers and novices, seem to be displayed by comparing Level 1 with Level 2 for solution Step 2 requiring diagram analysis.

Level 1 solution scripts, with no diagram errors in Step 2, had a record of 24% incorrect statements for Step 4 and 48% incorrect statements for Step 8. It can be seen, in Table 15, that Step 4 of Level 1 has less than half the incorrect statements as for Level 2, but Step 8 for Level 1 (45% incorrect) is marginally greater than the error tally for Level 2 (44% incorrect).

It may be seen that substantially increased errors of diagram omission, in Level 1, could have influenced the frequency of subsequent equation errors at Step 8 of Level 1. This could have been caused by the omission (hence lack of clarity) of diagram analytic detail.

Experience gained by analysis of solution steps in both the drawbridge and the jibcrane problems suggested the need for finer analytical detail, in studying student handling of diagram analysis. This led to construction of the Mechanics Vector Analysis Test, whose results are presented next. The purpose of this test was to obtain more specific insights to the problems that students have in vector analysis of mechanics diagrams.

RESULTS OF ANALYSIS - 1981 MECHANICS VECTOR ANALYSIS TEST

A total of 168 Year 12 physics students from Western Australian schools took part in the testing programme. Thirty separate diagram questions were presented. Each page of questions was designed to test a specific region of problem solving skills using diagrams. Each page of questions therefore, was analysed separately, for error occurrence. Details of step numbers and their solutions are shown on pages 194 to 199.

Page 1. Mechanics Vector Analysis Test

Diagrams on page 1 of the mechanics vector analysis test in Appendix A tested student ability to sketch the resultant vector of two component forces. A frequency distribution of errors in each problem solving step was determined. The results of that analysis are presented in Table 16 and Figure 36 respectively.

Table 16

Page 1. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency
Sample Size : N = 168

Step Number	Error Frequency	Errors (%)	Step Number	Error Frequency	Errors (%)
11	7	4	25	59	35
12	25	15	26	15	10
13	27	16	27	10	9
14	8	5	28	16	6
15	8	5	29	15	10
16	6	4	30	29	9
17	16	10	31	51	17
18	7	4	32	31	30
19	3	2	33	43	8
20	30	18	34	38	26
21	52	31	35	14	23
22	5	3	36	45	8
23	10	6	37	64	27
24	48	29	38	25	15

(Steps 0 to 10 were used to record administrative data.)

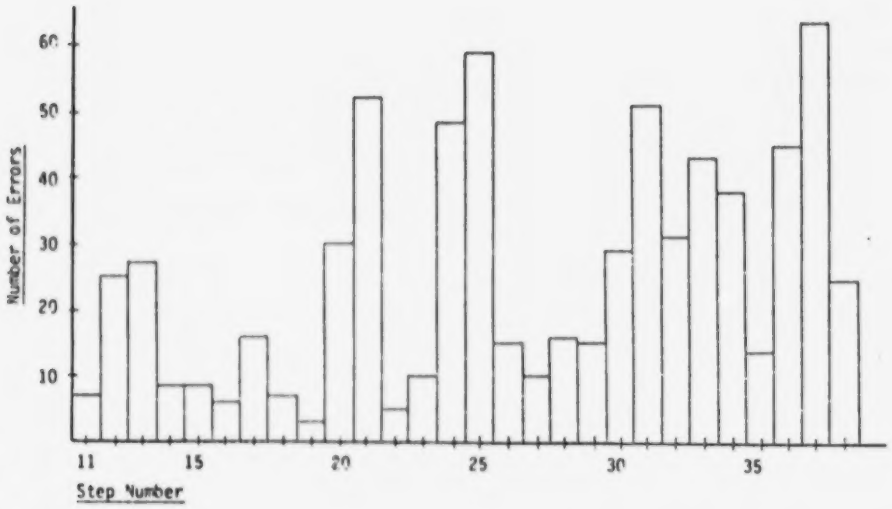


Figure 36

Page 1 - Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency.

Discussion of Page 1 Errors

The histogram in Figure 36, shows six peaks, indicating the several steps where students experienced greatest common difficulty. Cross checking with the test paper showed that one of the peak areas was probably a misleading result. Students had often inadvertently omitted diagram 6 on page 1 of the test paper. In the process of duplication, the diagram had become quite difficult to identify. Since omission was regarded as one form of error, it was decided that steps 31 to 34 should be disregarded when interpreting the graph.

Graph analysis identified the key error in student performance as involving the construction of vector magnitudes. These errors occurred at steps 13, 17, 21, 25, 29 and 37. Each of these steps (except 29) appeared as a peak in error frequency on the graph.

Step 29 required the construction of a triangle of forces. Apparently, most students were familiar with, and were competent in, the practice of drawing a closed triangle of forces as a basis for problem solving progress. This background would therefore have given a correct vector size regardless of student understanding of vector magnitude.

Students in general, performed competently in locating the line of action and direction of each resultant vector. This result was then considered in combination with error frequency for vector magnitude. That students were able to construct a correct line of action but then drew an unrealistic vector size perhaps suggests a reason why so many students, in the drawbridge and jib crane problems, arrived at equally unrealistic quantity values for their final answers. Further comment has been recorded in the Conclusions of Chapter 5.

From Figure 36 for Page 1 of the Test, it could be seen that students were not estimating size quantities that apply in a physical situation. The same error arose in the jib crane. For example, when lifting a load of 20,000N, arrival at an answer of 4,000N tension is obviously ludicrous. Apparently, students have successfully divorced their theoretical methods of calculation from the practical constraints of the system under tension.

Students had made no note that the answers they arrived at appeared unreasonably large or small. It is noted that these questions were not specifically asked. This suggests that they were, in fact, unaware of how to estimate a vector size. This is a vital means of checking a problem solution. This outcome will be discussed further under implications in Chapter 5.

Page 2. Mechanics Vector Analysis Test.

Diagrams on Page 2 asked students to distinguish freebody diagrams that were in equilibrium, from those that were not in a state of equilibrium. The analysis of error occurrence for each question was presented in Table 17 and represented graphically in Figure 37.

Table 17

Page 2. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency
Sample Size : N = 168

Step Number	Number of Errors	% Errors
39	4	2
40	5	3
41	4	2
42	53	32
43	12	7
44	10	6
45	45	27
46	9	5
47	17	10
48	4	2

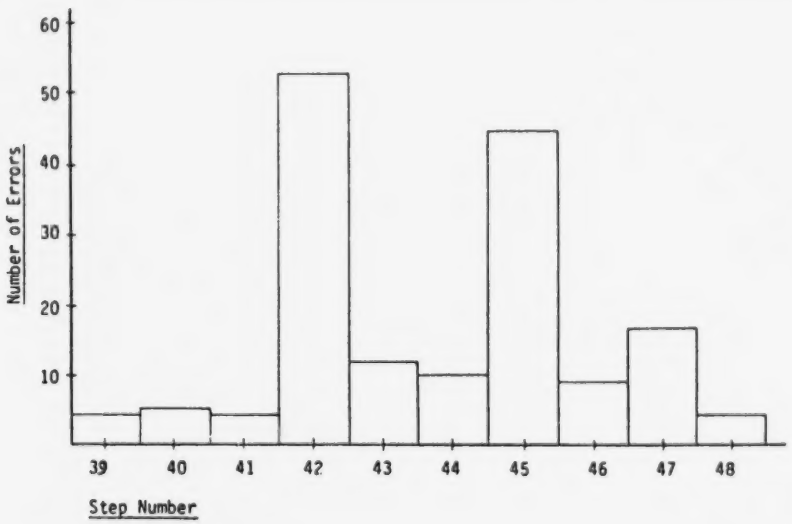


Figure 37

Page 2 - Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency.

Discussion of Page 2 Errors

Overall, students did well on this test page. As can be seen from the peaks in the graph of Figure 37, difficulties were encountered at Step 42 (31% incorrect) and Step 45 (27% incorrect). In each of these steps, a false resultant of two vectors was drawn in. Students failed to recognise that an equilibrium situation did not exist. Further comments have been made under Implications in Chapter 5.

The graphical analysis of page 2 (Figure 37) in the test showed that an appreciable fraction of students have failed to recognise an equilibrium situation in a set of interacting forces, for a specific case.

Page 3. Mechanics Vector Analysis Test

Diagrams on page 3 of the test paper called upon students to mark correct directional arrowheads on a freebody diagram in equilibrium. Analysis of the errors is presented in Table 18 and a graphical representation of the data follows in Figure 38.

Table 18

Page 3. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency
Sample Size : N = 168

Step Number	Number of Errors	% Errors
49	6	4
50	34	20
51	3	2
52	37	22
53	5	3
54	38	23
55	18	11
56	34	20

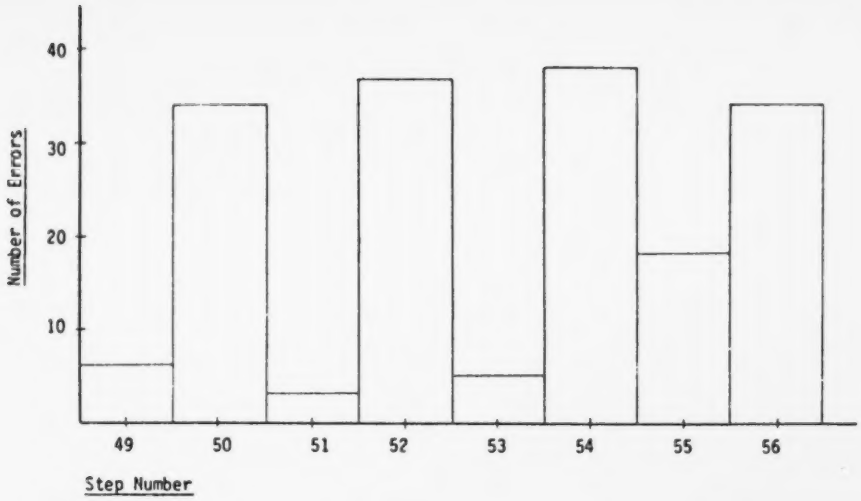


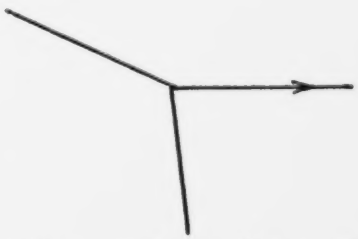
Figure 38

Page 3 - Mechanics Vector Analysis.
Solution Step Number and Error Frequency.

Discussion of Page 3 Errors

Student scripts, in general, correctly inserted force direction arrowheads when the vectors were drawn as illustrated in Figure 39 (a) below:

(a)



When the vector was moved to position (b) as follows:

(b)

Figure 39

Diagrams showing reorientation of vectors

students were then apparently unable to correctly interpret the force directions needed to set up a state of equilibrium.

From page 3 (Figure 38) of the test, it appeared that students could not interpret the necessary vector directions appropriate to handling similar difficulties in the jib crane problem. Here is a precise parallel to the vector difficulties that students encountered in the jib crane problem. This then, helps verify one type of difficulty that may have been confounding student attempts in the jib crane diagram.

Page 4. Mechanics Vector Analysis Test

Page 4 asked students for a description of the principles being applied on page 5. This detail became extraneous to the focus of the thesis and the results were therefore omitted from discussion.

Page 5. Mechanics Vector Analysis Test

This page displayed 5 exploded diagrams. On each of these diagrams, the student was required to show the origin, direction and point of concurrency of the three forces that kept the nominated body in equilibrium.

On this page, students were asked to apply vectors to a concrete equilibrium situation. Results for each of these problems have been tabulated and graphed separately, with a brief analysis of each diagram. References to the test paper in Appendix A will clarify the nature of each diagram.

The error tabulations for problem 1, page 5, are presented in Table 19 and is graphically represented in Figure 40.

Table 19

Page 5, Problem 1. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency
Sample Size : N = 168

Step Number	Number of Errors	% of Errors
57	12	7
58	133	79
59	133	79
60	45	27
61	26	16
62	25	15
63	9	5
64	35	21
65	35	21
66	11	6
67	34	20
68	34	20

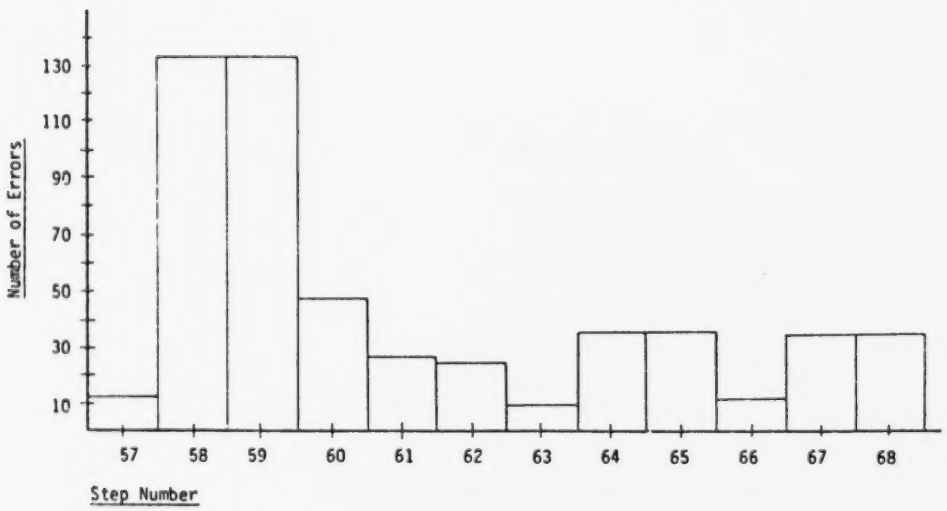


Figure 40

Page 5 - Problem 1. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency.

Discussion of errors in Problem 1, page 5

The error present in step 60 involves line of action of the weight of the ball. The size of this error frequency could be a false indicator. In retrospect, it was felt that the diagram had been somewhat misleading. It had not been made clear in the drawing that the apex of support had been tilted to one side. Consequently, many students drew the line of action through the apex rather than vertically downward.

Incorrect location for the point of application of each line of action was the most common error (79% incorrect for steps 58 and 59). The originating position was often incorrectly placed on the supporting, rather than on the supported surface. There is a very fine line of diagrammatic distinction here, which would probably not affect the results of calculation.

The error tabulations for Problem 2 on page 5 are presented in Table 20 and are graphically represented in Figure 41.

Table 20

Page 5, Problem 2. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency
Sample Size : N = 168

Step Number	Number of Errors	% Errors
69	10	7
70	141	84
71	101	60
72	13	8
73	58	34
74	12	7
75	3	2
76	86	51
77	13	8
78	7	4
79	26	16
80	51	30

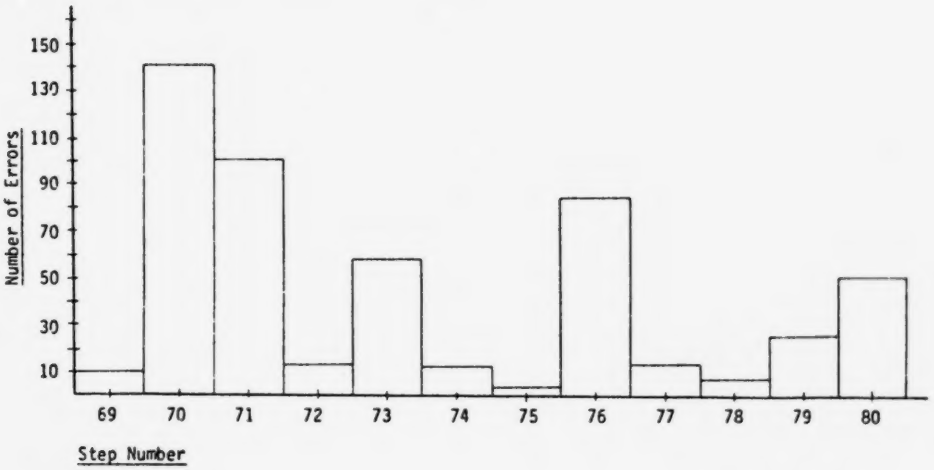


Figure 41

Page 5 - Problem 2. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency.

Discussion of errors in Problem 2, page 5

Many solutions in problem 2 did not locate the point of application upon the supported surface. Instead, 84% of students in step 70 located the point of application, in error, at the supporting surface. In general, reaction force had several errors associated with it, especially the incorrect line of action and omission of its point of intersection with the other forces.

A significant error involved the failure by students to extend the tension vector, by dotted line from its point of application to the point of concurrency. Errors made in step 80 arose when students failed to recognise that all forces in equilibrium extend their lines of action through a common point of concurrency. Further details on this point will be considered in the recommendations of Chapter 5, under the strategy of diagram analysis : stage 5, called Vector Extension.

Error occurrence for problem 3 on page 5 of the test is presented in Table 21 and is graphically represented in Figure 42.

Table 21

Page 5, Problem 3. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency
Sample Size : N = 168

Step Number	Number of Errors	% Errors
81	22	13
82	72	43
83	134	80
84	5	2
85	28	17
86	122	73
87	4	3
88	22	13
89	56	33
90	129	77
91	67	40
92	141	84

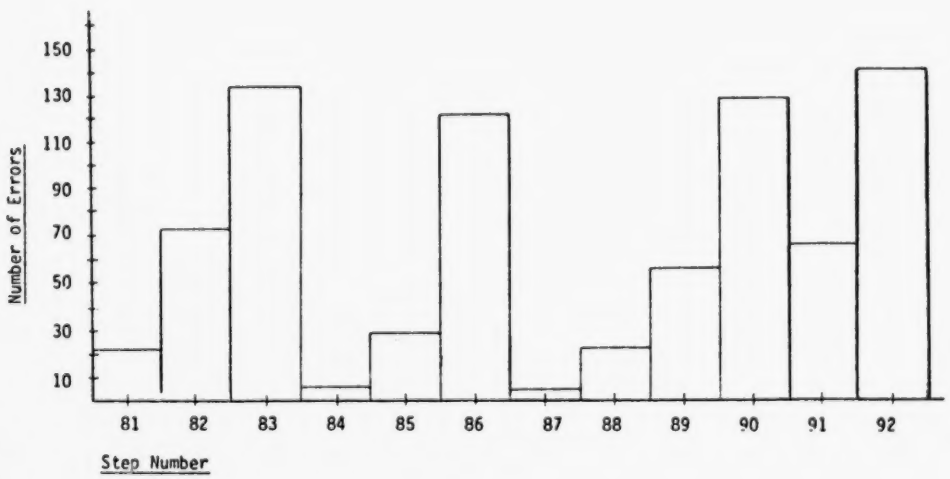


Figure 42

Page 5 - Problem 3. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency.

Discussion of errors in Problem 3, page 5

Students' performances, in handling force at the pivot in this diagram, produced an exceptionally high error occurrence. In step 83, 80% of students did not correctly originate this vector or the direction in which its force was acting. Students could obviously see the concrete force of tension in the supporting cord. They were also able to visualise the weight through its centre of gravity. Apparently, however, they could not see that a reaction at the pivot was necessary to keep the system in equilibrium.

Error occurrence for problem 4 on page 5 of the test is presented in Table 22 and is graphically represented in Figure 43.

Table 22

Page 5, Problem 4. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency
Sample Size : N = 168

Step Number	Number of Errors	% Errors
93	18	10
94	53	32
95	53	32
96	2	1
97	28	17
98	28	17
99	6	4
100	14	8
101	14	8
102	107	64
103	57	34
104	58	34

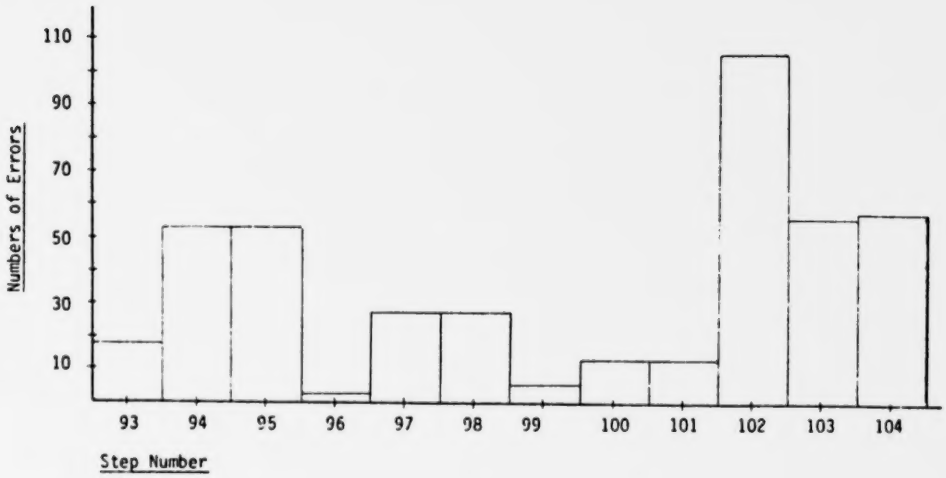


Figure 43

Page 5 - Problem 4. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency.

Discussion of errors in Problem 4, page 5

Problem 4 gave the best student performance results. This is probably due to the nature of the diagram, which gave a strong hint as to the line of action of each force. Students were, in fact, dealing here with 'concrete' forces rather than with more abstract reaction forces of previous problems on this page.

Once again, the step causing greatest difficulty involved extending the vector line of action by a dotted line from the centre of gravity up to the point of concurrency. In Step 102, 64% of students failed to extend that vector to its point of concurrency. This difficulty has been accounted for in the detail of stage 5 of the recommended strategy for diagram analysis. (See Chapter 5)

One of the difficulties in scoring this particular problem involved the way in which students drew their vectors. Many lines were so faint that they could barely be seen. Perhaps a dotted diagram or a

fainter diagram might have elicited a more definite response in student attempts.

Error occurrence for Problem 5 on page 5 of the Test is presented in Table 23 and is graphically represented in Figure 44.

Table 23

Page 5, Problem 5. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency
Sample Size : N = 168

Step Number	Number of Errors	% Errors
105	14	8
106	150	89
107	132	79
108	14	8
109	64	38
110	15	9
111	10	6
112	89	53
113	28	17
114	12	7
115	29	17
116	30	18

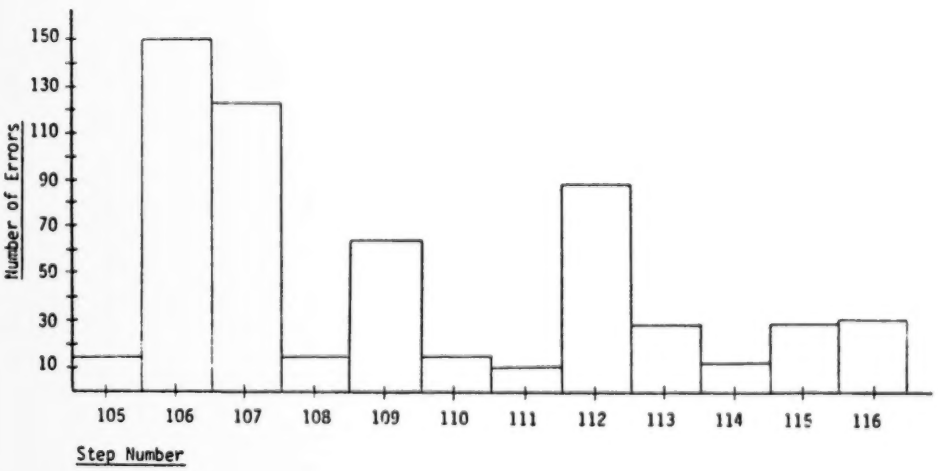


Figure 44

Page 5 - Problem 5. Mechanics Vector Analysis Test.
Solution Step Number and Error Frequency.

Discussion of errors in Problem 5, page 5

Problem 5 was of a nature similar to problem 2, page 5. Once again, an exceptionally high number of errors (89%) occurred in step 106, by misplacement of the point of application of the reaction force, on the supporting plane, rather than on the supported plane. Difficulties were also experienced in placing and in locating a point of application for the spring.

Errors occurring at steps 109 and 112 were associated with reaction force of the sloping plane. Students found great difficulty in positioning correctly the line of action and the arrowhead direction for that reaction force.

SUMMARY

In general, students handled the weight vector correctly but mishandled the other two vectors involved in the equilibrium.

Students seemed to be unaware of the distinction between the resultant of a number of vectors and the equilibrant of those vectors. Students did not seem at ease with the location of vectors. The relocation in the orientation of the problem was a confusing factor for students.

Students generally, handled concrete examples of equilibrium situations well. But they were not successful in representing the more abstract forces of reaction at planes and pivots.

Students were in general, unable to draw the point of concurrency. They seemed in fact, to be unaware that for an equilibrium situation, all vectors intersect at a common point of concurrency.

CHAPTER 5

SUMMARY, CONCLUSIONS, IMPLICATIONS AND RECOMMENDATIONS

SUMMARY OF THE STUDY

Tertiary admission examination attempts by students in the 1978 and 1979 physics papers were used as an information base, to identify and classify student difficulties in selected mechanics problems. The most common student difficulty was identified as diagrammatic visualisation of each problem statement and its solution steps.

A further study examined in detail the error types in student performance on diagram analysis. The outcome of this research was the evidence that student problem solvers had many simple as well as complex difficulties in illustrating the mechanics problem statement and in analysing its vector components. These findings form the basis for implications and recommendations which follow.

CONCLUSIONS OF THE STUDY

As part of the research design and methodology, a series of hypotheses were presented to guide the direction of this thesis. These general propositions are now re-examined, in the light of results presented in the previous chapter.

Hypothesis 1:

That specific difficulties experienced by students, in solving complex mechanics problems, may be distinguished, identified and classified.

As a result of analysing student drawbridge and jibcrane solution scripts, many difficulties were distinguished and identified. Discussions of these difficulties are recorded with examples further on in this chapter. These difficulties were classified under five categories of problem solving difficulty: comprehension, translation, analysis, mathematics and synthesis, as discussed further on.

Hypothesis 2:
That some difficulties in mechanics problem solving are common to all physics students.

Across all five categories of error classification and throughout each level of student ability, difficulties of diagram analysis were a common obstacle to correct solution progress. This was the common difficulty thoroughly analysed in the Mechanics Vector Analysis Test. The findings therefore, may not become appropriate to problems where diagrams are not so relevant.

Hypothesis 3:
That the use of diagrams to help solve mechanics problems can give students a greater chance of processing a successful solution.

In both drawbridge and jibcrane problems, many errors in student solutions occurred as a result of diagram omission or where a freebody diagram was not drawn in isolation from the a physical situation diagram. The major conclusion of this finding is that many student problem solvers were unaware of how to draw diagrams accurately.

The follow-up research gave evidence that many students were unable to convert the diagrams of physical situation into a diagram of relevant vectors.

Students did have the time constraint of examination conditions for the drawbridge and jibcrane problems. So this could have reduced the time needed for presenting more than one diagram.

In the follow-up research, time was not a valid constraint. Each student was given sufficient time to complete every section. From these results, it seems that many students had not learned or did not understand how to use diagram analysis as an aid to solution process. It was the absence of subsequent diagrams of isolated vectors which was a major obstacle to correct solution processing. The findings are strongly supportive of the hypothesis. They could not however, provide verification. This would come only as a result of a pretest followed by the explicit teaching of the methods of diagram analysis. A post test could then verify or disclaim the third hypothesis. This hypothesis must therefore, remain as a prediction yet to be tested.

From the analysis and interpretation of results in Chapter 4, the various error categories have been related to the statements of hypothesis. This had led to relevant conclusions. The general conclusion is that poor diagram analysis is a major influence on mechanics problem solving competence among Tertiary Admission examinees in Western Australia. This conclusion also serves as a critical justification for the implications and recommendations which follow.

IDENTIFICATION OF ERROR TYPES RELATED TO DIAGRAMS

To illustrate the different categories of errors identified, only the drawbridge example has been used here, in order to avoid confusion. The results in Chapter 4, however, will show that the illustrations are equally relevant to the jibcrane solution scripts.

Error Type 1 Presentation of a Physical Diagram

Initial errors occurred where students were unable to correctly draw the diagram of the physical situation. An alternate opinion of the nature of this error was that students could not visualise the problem statement. They therefore would have had only a vague mental picture on which to base a translation to the physical diagram.

All students from Level 1 and Level 2 drew some form of diagram. It is possible therefore, that students who were unable to draw a diagram, may have omitted the question for that reason.

Error Type 2 Problem Component Location

Errors in location of forces occurred where students omitted to indicate the correct position of given information on the physical diagram.

Steps requiring students to locate component information, were:

Step 2 drawbridge length of 60 m.

Step 3 T_R diagram component vector.

Step 4 T_R contact with drawbridge at 40 m.

Table 24
Student omissions for Steps 2, 3, 4 of the Drawbridge Solution

Step Number	Level 1	(%)	Level 2	(%)
2	8	13	13	13
3	7	11	13	13
4	2	3	3	3
Total errors	17		29	
Sample size	65		103	

For Level 1 students, errors of this type represented 27% (Table 7) of all errors made in part (i) of the solution method. For Level 2 students, errors of this type seemed to lead to a rapid increase of subsequent mistakes in steps further on. When relating these errors to the problem statement, it seems that careful reading and sectioning of the questions could have helped avoid most of these difficulties in transposing given information. This seemed to be a matter of problem statement comprehension.

Error Type 3 Isolation of Freebody Vectors

Error occurred where students were unable to transform the physical situation into a freebody isolation. This process of diagrammatic sequence development, demands the ability to recognise and separate all the relevant forces acting on the body in the physical diagram (Meriam, 1968). Analysis and translation are the problem solving operational processes involved. The process of mental transformation between problem statement, its visualisation, the physical diagram and freebody isolation seem to be relevant considerations in error occurrences of this type.

The following steps involved the isolation of a freebody diagram:

Step 6	T_Y diagram	
Step 10	distance	PC = 30 m
Step 11	height	DP = 40 m
Step 12	distance	PB = 40 m (distance to T_Y).

Table 25

Student Error Frequency for Steps 6, 10, 11 and 12 of the Drawbridge Solution

Step Number	Level 1	(%)	Level 2	(%)
6	7	11	18	18
10	1	2	18	18
11	1	2	3	3
12	0	0	2	2
Total errors	9		41	
Sample size	65		103	

The main area of difficulty seemed to have arisen from student omission of orthogonal components in the vector diagram. Orthogonal vector resolution helps clarify the line of action of each operating force and its appropriate components. It also assists substitution into moments equations. A diagram consisting only of these analysed components, could also have indicated ways to solve the problem. For example, such a diagram could have suggested whether to use moments or the addition of horizontal and vertical forces, in evaluating the pivot compression force of the drawbridge.

Error Type 4 Properties of Triangles

Errors occurred when students failed to apply simple trigonometric and geometric laws of triangles. Correct use of pythagoras rule, sine and cosine ratios, sine and cosine laws is prerequisite to progress in conversion of a vector diagram to a mathematical equation.

For example, in the drawbridge problem, solution steps requiring use of triangle properties were as follows:

Step 7 $T_Y = T_R \cos \theta$

Step 13 $\theta = 45°$

Step 40 $\tan \theta = \frac{T_V}{T_H}$

Step 41 $\tan^{-1} \frac{T_V}{T_H} = 18{\cdot}43°$

Step 43 $T_p = [(T_V)^2 + (T_H)^2]^{\frac{1}{2}}$

Solution synthesis ideally requires a logical linking of component properties, rules and values. Steps 40 and 43 showed a large number of errors in which students gave evidence of difficulties with logical synthesis, when using diagrams and diagrammatic rules.

IDENTIFIED CATEGORIES OF PROBLEM SOLVING DIFFICULTY

In scoring solution attempts, many points of error were noted. Prior to level separation however, no specific trend or occurrence of specific error type had been clearly identified.

After the three ability levels were separated, a totally different pattern was observable. From the separate collective records of Levels 1 and 2, the occurrence of some specific errors and their trends became immediately evident even before graphing commenced.

The most obvious area of initial confusion for students occurred within the problem statement itself. Most initial errors seemed to be related to a lack of organisation. Students did not seem to know what to do with the given information. There was little attempt to summarise the information supplied. Perhaps students did not have the time to do this. Positioning of given information in an organised diagram was often at variance with the values supplied in the problem statement.

Comprehension

Students found difficulty in building a picture or drawing an actual diagram of the drawbridge from the given information. The difficulty was classified under physics comprehension.

Comprehension Difficulties

Difficulties encountered in the drawbridge and jibcrane were:

1. Extracting the given details and values, e.g., labelling the diagram with given values.
2. Summarising the given information, e.g., in a properly labelled diagram.
3. Positioning the given information correctly on the visualised diagram, e.g., locating the centre of gravity of the drawbridge.
4. Visualising the physical situation, that is, drawing the drawbridge diagram, as described in the problem statement.
5. Comprehending the need for reasonable assumptions, e.g., assuming the correct position for the centre of gravity of the drawbridge and showing that vectors have a specified direction, by the use of arrowheads.

Translation

Generating a correct physical diagram, by an accurate translation from the given information of the problem, was an early observation of initial student difficulty. Transforming that physical diagram into a freebody diagram, often produced a compound difficulty because of previous incorrect positioning or labelling errors in the physical diagram.

Transformation to a statement of general principle from the diagram attempt, in many cases, was not stated or was unclear. The subsequent statement of mathematical equation, often without any prior reference formula, suggests formula "plugging" as a student problem. It also emphasised student insensitivity to a logical sequence of explanation.

Translation Difficulties

The points of error in translation are recorded as follows.

1. Drawing a physical diagram from the given information of the problem; a typical error involved the incorrect placement of chain contact at the end of the drawbridge.
2. Translating the given information to lengths, angles and points of contact within the physical diagram; for example, the contact points of the chain, with drawbridge and the wall, were each at 40 metres from the pivot, requiring angles of 45°, by simple trigonometric implication.
3. Transforming the physical situation to a freebody diagram of vectors. For example, Steps 6, 14, 30, 39 and 42 were often omitted from the drawbridge solution.

Analysis

Most students gave evidence of understanding the need for a diagram by sketching a drawbridge. To recognise, separate and extract specific values, components and principles from the problem statement, was an area where many diagrammatic and mathematical misinterpretations had their origin. Analysing component phrases from the general problem statement, was often inaccurate, with values and vectors being transposed incorrectly.

Difficulties With Analysis

1. Extracting essential vectors from the physical diagram to form a freebody isolation, e.g., drawing and labelling Steps 6, 10 and 14.
2. Separation of specific vectors into orthogonal components, e.g., Steps 18 and 30.
3. Recognising and using implied assumptions to assist vector analysis.

   Examples: properties of isosceles triangles
   properties of right triangles
   assumptions about position of centre of gravity in a "uniform beam" (Step 10).
4. Analysing and labelling the correct direction of a vector within the physical and freebody frame of reference, e.g., Steps 4, 6 and 8.
5. Extracting individual values and properties from the general mass of information provided in the problem.
6. Descriptive labelling of the various parts of a diagram, e.g., Step 6.

Mathematics

Handling the simple processes of mathematical manipulation, substitution and simplification were substantial and consistent sources of error and omission. Sixty students, out of 103 from Level 2, substituted values incorrectly into the first equation of moments, for Step 14 of the drawbridge solution. Forty-nine of these students failed to simplify that same substitution correctly. Thirty-two of those students could not isolate the variable in that statement correctly. Nineteen, out of the 103 Level 2 students, could not arrive at a correct final answer to that momemnts calculation.

Mathematical Difficulties

1. Properties of isosceles triangles, e.g., Steps 12 and 13.
2. Properties of right triangles, e.g., Step 18.
3. Pythagoras rule, e.g., Steps 42 and 43.
4. Properties of similar triangles, e.g., Step 39.
5. Trigonometric ratios of triangles, e.g., Steps 7, 19 and 40.
6. Substitution for values in formulae, e.g., Steps 15, 19, 32, 37 and 44.
7. Isolation of an unknown variable, e.g., Steps 7, 16, 20, 33 and 38.
8. Crossmultiplication, e.g., Steps 16, 33, 38.
9. Simplification, e.g., Steps 17, 20, 34, 39, 45.
10. Significant figures, e.g., Steps 20, 45.
11. Expression for angle of direction, e.g., Step 45.

Synthesis

The basic process of synthesis involves drawing together, sequencing and linking the analysed components of a problem's solution.

Omissions in drawing the threads of analysis together is a difficulty which demands competence in formal reasoning (Meriam, 1968), and perhaps even a sense of creativity. The whole field of invention and creative thinking is at present in a general state of academic uncertainty (Poincaré, 1960). It is therefore likely that students will make errors of commission and omission, when even the guidelines of experts are somewhat nebulous (Guildford, 1967; Adams, 1974).

Of course, the pressure of examination too, can force the commission of errors and the omission of logical steps. This investigation however, has concentrated on the synthesis of diagrams and related formulae.

Difficulties of Synthesis

A list of difficulties related to synthesis is detailed below.

1. Drawing a vectorgon from a set of analysed vectors, e.g., Steps 6 and 42.
2. Finding a resultant by using trigonometric rules, e.g., Steps 39, 42.
3. Recognising and applying pythagoras rule to a vector situation, e.g., Steps 19, 43.
4. Two stage formula application, e.g., Steps 40, 43.

Examination of the points of error in each of the divisional categories revealed one common error type above all others.

This lay in the area of error and omission related to diagrammatic visualisation.

Among errors of comprehension were included locating and positioning of given information on the visualised diagram. Errors of translation involved drawing a physical diagram from the problem statement and transforming the physical diagram into a freebody diagram. Errors of analysis mentioned the extraction of essential values and vectors from the physical diagram to form a freebody isolation. It also emphasised the recognition and use of implied vector assumptions, properties and appropriate labels. Mathematical difficulties involved properties of triangles and the expression for angled direction. Errors of synthesis noted the failure to draw analysed vectors together, thereby reducing them to a final resultant force.

From the above summary may be seen a collection of diagrammatic items of error. Each item related to the application of diagrammatic visualisation in physics problem solving. For this collective reason, it was seen that research work into diagram analysis might usefully provide a better understanding for teaching strategy in each identified category of problem solving difficulty.

In summary, problem solving difficulties were interpreted under the following categories: comprehension, translation, analysis, mathematics and synthesis. These classifications have provided a useful parallel with an accepted taxonomy of behavioural objectives (Bloom, 1970).

It may therefore, be reasonable to anticipate that a significant improvement in diagrammatic visualisation might positively affect every category of problem solving performance. It is the intention of this study to aim at the point of strategy development which will suggest ways to improve student performance in all categories of diagrammatic visualisation.

Comprehension of the problem statement included difficulties of locating given and assumed information at the correct position of the drawbridge diagrams. For example, the most competent problem solving scripts showed no difficulty in locating the assumed position of the centre of gravity for the drawbridge. In the less competent performance level however, 12% of these students failed to correctly position the centre of gravity vector for the drawbridge. Nearly as many gave the vector an incorrect value.

These errors are, most likely, evidence of difficulty with comprehension of the problem statement. A proper understanding of the drawbridge problem statement includes drawing out assumed knowledge. It requires a clear comprehension of what is implied, as well as an understanding of what has been expressed in writing. The conclusion drawn is that many students had difficulty with the comprehension of implied diagrammatic concepts.

Accurate translation from written information of the problem statement helps generate a diagram of the physical situation (a physical diagram). Accuracy of translation tended to be a source of student difficulty. Transforming that physical diagram into an isolated freebody diagram was often a continuing source of student error.

One error between the written statement and physical diagram often led students into another error when translating further into the freebody diagram. Omission of this further translation was also a common error occurrence. Time constraints and examination pressure would have been severe limitations, from which many omission errors could have originated.

One conclusion arising from such translation errors was that student performances tended to produce equation statements without any diagrammatic evidence in support of such a relationship. Such action sometimes seemed to be an almost random attempt to substitute values into an often unstated relation or equation, in the hope of arriving at the right answer. The second conclusion is that difficulty in translation arose as a result of student insensitivity to a logical sequence of explanatory steps.

Difficulties of diagram analysis included errors when drawing and labelling horizontal and vertical vector components. The major conclusion here is that students had difficulty in extracting essential component vectors from the physical diagram to form a freebody isolated vector diagram.

Mathematical manipulation errors occurred in both drawbridge and jibcrane solution attempts. Handling simple processes of cross multiplication, substitution and simplification were substantial and consistent sources of error and omission. A list of mathematically grounded errors has been compiled and presented in the section on Drawbridge Error Analysis in Chapter 4. It is concluded that the generally low level of mathematical ability is a serious obstacle for many of the less competent students.

Because diagram analysis was selected as the most common student difficulty, the statistical details of mathematical errors have not been fully investigated. Incorrect labelling of component vectors along with errors in calculating their values, was however, a common source of mathematical error noted as part of diagram analysis.

IMPLICATIONS

This section begins by considering the significant conclusions of the study and relating them to implications for physics education.

In the Mechanics Vector Analysis Test, it was shown that students encountered many difficulties in the process of transforming a physical situation diagram into the more abstract presentation of a component vector diagram. Similar diagram difficulties and omissions were noted in student attempts to find a resultant for a number of vectors. Students therefore, find difficulty in their analytic and synthetic approaches to mechanics problem solving.

The large and common difficulty students encountered with vector diagrams in mechanics problem solving has several implications for physics education. The implications are that students do not know how to analyse a diagram and that they are unaware of the use of diagram analysis for logical solution sequence. It also appears that diagram analysis needs greater emphasis as an integral part of problem solving teaching strategy.

The results of separate error analysis for levels 1 and 2 have drawn two conclusions. There are distinct and characteristic differences

in the types of errors first encountered by each level, as detailed in Chapter 4. There are also errors common to each level across each identified error category.

The implications for physics education are that there are distinct subgroups of competence within problem solving performance. This finding is not new and is well recognised. That there are distinct differences in error types first encountered, does imply however, the need to consider separate teaching strategies appropriate to student competence.

Significant among the implications from the Mechanics Vector Analysis Test was the implied demand for a clearer explanation in the teaching of diagram analysis. Strategy development of greater detail in the sequencing of diagram development becomes the conclusive recommendation of this study. A detailed example of such a strategy applied to the drawbridge will follow.

RECOMMENDATIONS

Introduction to Diagram Analysis

The chief recommendation of this thesis calls for a thorough program of practice in vector diagrams, as a specific method for improving mechanics problem solving competence among upper secondary physics students.

It is suggested that a properly oriented program of diagram analysis might be conducive to the more spontaneous generation of inventive, creative and intuitive perceptions of correct concepts in mechanics problem solving. This direction may be seen as an ultimate aim of physics education.

This study has drawn upon the analytical evidence of Chapter 4 to show that student misuse and omission of diagrams plays a major role in the difficulties encountered by student physics problem solvers.

As will be seen in the example to follow, the method chosen has two dimensions: detailed diagram analysis and its integration with an orderly and disciplined sequence of student reasoning.

Mechanics Problem Solving Strategy

Diagram analysis purports to develop a logical sequence of visual explanatory stages in the mechanics problem solving process. The example following separates nine stages of diagrammatic development. Each stage can be applied or omitted by a problem solver according to expertise and insight. For a novice however, it is suggested that each stage needs to be presented. It also needs to be seen as merging into the next stage - and applied fully in worked examples before committing the student to a similar problem.

Diagram analysis is now presented as a summary set of nine separated, named, development stages, with a brief descriptive analysis of each stage. Following the summary, a detailed

application of each stage in diagram development will follow, using the 1978 drawbridge problem. In each stage, an organised set of steps will be listed to illustrate possible supplementary teaching strategy.

These steps are designed to help students become gradually more independent in their progress toward each next stage of diagrammatic development.

How may this strategy be taught in a classroom situation? The answer seems to be related to the mode of teaching used. A blackboard presentation might take a long time. Overhead projections of one section at a time might allow quick progress from one step to the next. A simple set of duplicated worked examples would permit every student to follow each step at an individualised pace.

Some points of teaching strategy recommended are now listed briefly. A teaching strategy may best take advantage of the finding in this study by:

1. stating every step using model examples;
2. avoiding omission of assumed knowledge steps;
3. cumulative diagram development;
4. small step translation;
5. expressly teaching, discussing and explaining the purpose of each diagram;
6. providing related hints and questions to assist guided discovery;
7. providing specific step labels as an aid and a guide to memory.

STAGE SUMMARY OF DIAGRAM ANALYSIS FOR BODIES IN EQUILIBRIUM, UNDER THE ACTION OF THREE NON PARALLEL FORCES

Stage 1 - Problem Statement

This is a written description. It requires application of English comprehension, physics analytic comprehension, and the extraction of a summary information analysis.

Stage 2 - Visualisation

This is a mental image. It requires a questioning technique, instructive training, practice, a variety of visual experience, and imagination.

Stage 3 - Physical Diagram

This is a written diagram of the physical or mechanical situation. It requires a faithful and accurate translation from the mental image to the diagram being drawn.

Stage 4 - Freebody Isolation

This stage of diagram analysis gathers all the essential forces acting upon the body in question and isolates them from all other bodies or physical props in a diagram on their own.

Stage 5 - Vector Extension

This stage overlays the freebody forces upon a graphical reference frame. It requires extension of the lines of action of each force, until they intersect at a common point of concurrency. The purpose of vector extension is to utilise the principle that three coplanar forces acting on a body in equilibrium must be concurrent.

Stage 6 - Vectorgraph

This stage of diagram analysis requires the isolation of sectional vector groups upon a graph axial reference frame. It may be convenient to have more than one vectorgraph in order to deal separately with two vector groups.

Stage 7 - Orthograph

This stage further analyses the vectorgraph. It requires the representation of the vertical and horizontal components of forces from the vectorgraph. If there is more than one vectorgraph, then it is probably appropriate to have separated, matching orthographs.

Stage 8 - Solution Stage

This stage requires the aplication of conditions for equilibrium. It takes into account the use of moments about relevantly selected points, which will reduce the number of unknowns to a minimum.

Stage 9 - Vectorgon

This stage utilises the principle that, when drawn to scale, forces in equilibrium will form a closed polygon. It requires the construction of that closed equilibrium, polygon of forces, by scale diagram. Pythagorean and trigonometric properties of triangles may then be applied to find required vector values, for the case of three forces in equilibrium.

DIAGRAM ANALYSIS APPLIED TO THE DRAWBRIDGE PROBLEM

STAGE 1 - PROBLEM STATEMENT

The problem statement stage requires reading, understanding of terminology, analytical comprehension of sectional properties or measured values, and a (written) information summary analysis.

PROBLEM STATEMENT

A castle drawbridge of 2×10^3 kg is 60 m long. It is pivoted at one end at the bottom of the castle wall and it can be raised by a chain that is connected to the drawbridge 40 m from the pivot and to the castle wall 40 m above the pivot. When the tension in the chain is just sufficient to start raising the drawbridge from the horizontal position, calculate

i) the tension in the chain;

ii) the total force exerted on the pivot.

Steps for Student Progress

1. Read the question, building a mental image as you go through each subsection.
2. Read the question, making mental separation of each component description or measured value.
3. Write an information summary analysis:

Given	Castle	chain contact 40 metres up the wall.
	Drawbridge	length 60 metres; chain contact at 40 metres pivoted at base of castle wall.
Required to Find		i) chain tension force; ii) pivot compression force.

STAGE 2 - VISUALISATION

The visualisation stage requires formation of the relevant mental image of the problem situation.

Steps for Student Progress

Ask relevant questions: what does a drawbridge look like? Look at the various parts of that mental picture. Can you name some of the parts? What might the parts look like physically? Can you clarify the picture by using parts of the problem statement? What parts of the drawbridge are very clear? Do you understand how a drawbridge works? In your mind, can you raise the drawbridge? Can you lower it again? What raises and lowers this drawbridge? Where are the chain attachments to raise and lower the drawbridge? Where is the hinge (or pivot) of the drawbridge? Where is the chain pulling from? How long is the drawbridge? How heavy is the drawbridge? What does the drawbridge weigh? Where does the weight act?

As can be seen, there are many questions by which a picture building strategy may be assisted, clarified and analysed. Techniques of questioning may take on a more disciplined approach if the student is aware of what to do. Awareness should originate

logically from a teaching strategy; that is, the student should be shown, by example, how to develop this questioning technique and picture building process. This may then become a learning experience, by listing and comparing the most helpful types and quality of questions, which assist the correct picture to emerge.

MENTAL IMAGE

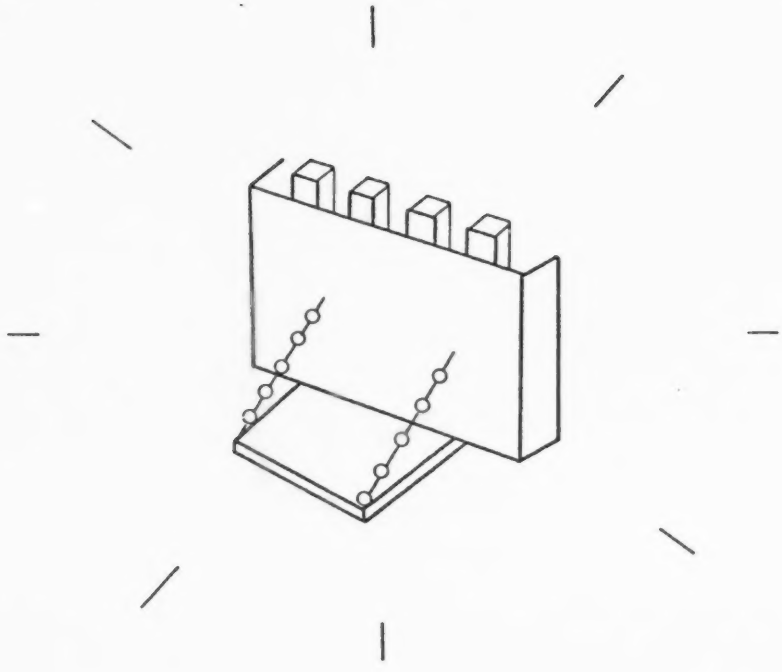


Figure 45

Drawbridge: Mental Image

STAGE 3 - PHYSICAL DIAGRAM

The physical diagram stage requires a faithful and accurate translation from the image in the mind to a diagrammatic visualisation on paper.

Steps for Student Progress

1. Sketch a diagram from the mental image.
2. Refer to the analysed information summary.
3. Locate the easily positioned, measured values on the diagram.
4. Does the diagram fit the facts?
5. If not, adjust the diagram.

PHYSICAL DIAGRAM

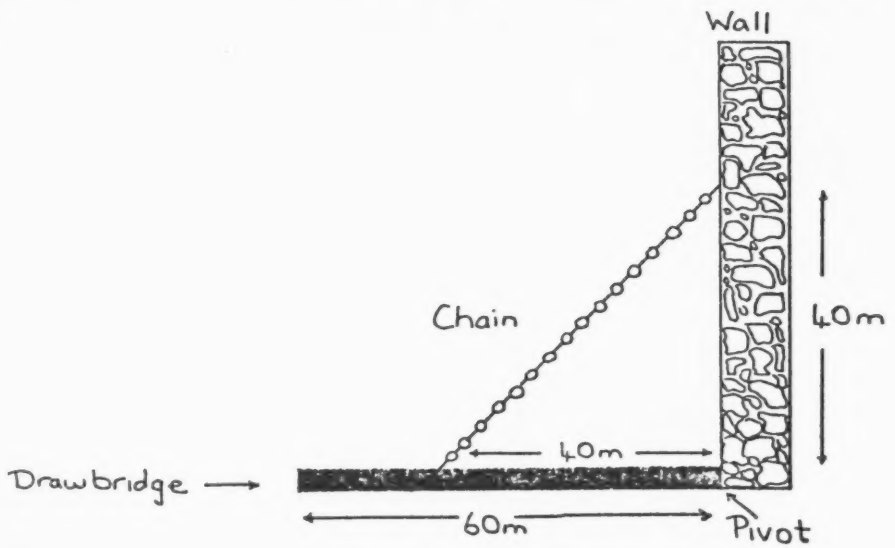


Figure 46

Drawbridge: Physical Diagram

STAGE 4 - FREEBODY ISOLATION

The freebody stage requires that all essential forces acting on the body be isolated from all other bodies so that a complete and accurate account may be taken of all forces acting on that body.

Steps for Student Progress

1. Identify and name the forces.
2. Draw on a new diagram the line of action of each force parallel to and in the same positional relation with its physical prop.
3. Do not draw any physical props.
4. Label the forces with appropriate names and symbols.
5. Label the vector directions with arrowheads.
6. Check the equilibrium demand that moments are not all going in the same rotational direction, with respect to a point.
7. If not so, correct one or more arrowheads to fit the factual force direction.
8. Does the freebody fit the factual information?
9. If not, adjust the freebody.

FREEBODY ISOLATION

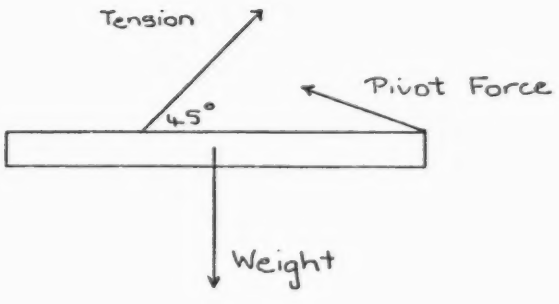


Figure 47

Drawbridge: Freebody Isolation

STAGE 5 - VECTOR EXTENSION

The vector extension stage utilises the principle that the lines of action of three forces in equilibrium must be concurrent. The intersection of the vectors involved in the drawbridge problem will therefore occur at a single point of concurrency, either inside or outside of the freebody diagram.

Steps for Student Progress

1. Be aware of the facts about the principle of concurrency.
2. Transfer the force vectors to a new diagram.
3. Maintain the same diagrammatic relationship between the forces.
4. Place a ruler along each line of action.
5. Extend each line of action with a dotted line.
6. In an equilibrium situation, the lines of action of the forces will always intersect at a common point, called the point of concurrency.
7. Show all distances and angles.

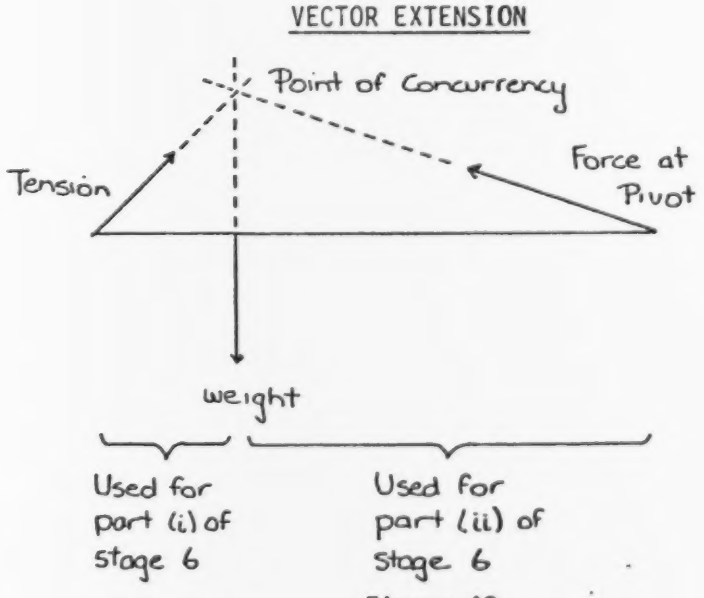


Figure 48

Drawbridge: Vector Extension

STAGE 6 - VECTORGRAPH

The vectorgraph stage requires isolation of vectors on a graph reference frame.

Steps for Student Progress

1. The student should be made aware that a method of solution is now present.
2. Seek this method from the diagram.
3. It should be noted that, for a body to be in equilibrium, there are two necessary conditions.
4. The sum of the moments about any point in the plane of the figure must have a zero value. In other words, the net torque about any point in the plane of the figure is zero.
5. The sum of the horizontal forces has a net zero value and the sum of the vertical forces has a net zero value.
6. Use either or both of these conditions to help find a solution.
7. Note that there are three vectors maintaining equilibrium in this problem: the drawbridge weight, the chain tension and the pivot reaction.
8. The weight (drawbridge centre of gravity) is already known.
9. The wanted vectors are the chain tension and the pivot reaction.
10. Find the physical connection between each wanted vector value and the weight or centre of gravity of the drawbridge.
11. Draw this physical connection on a graph reference frame.

VECTORGRAPH (i)

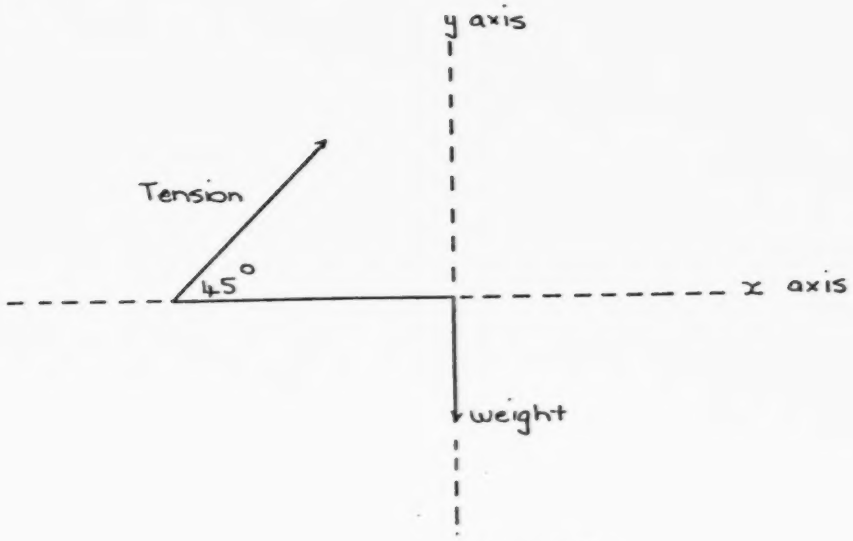


Figure 49

Drawbridge: Vectorgraph (i)

VECTORGRAPH (ii)

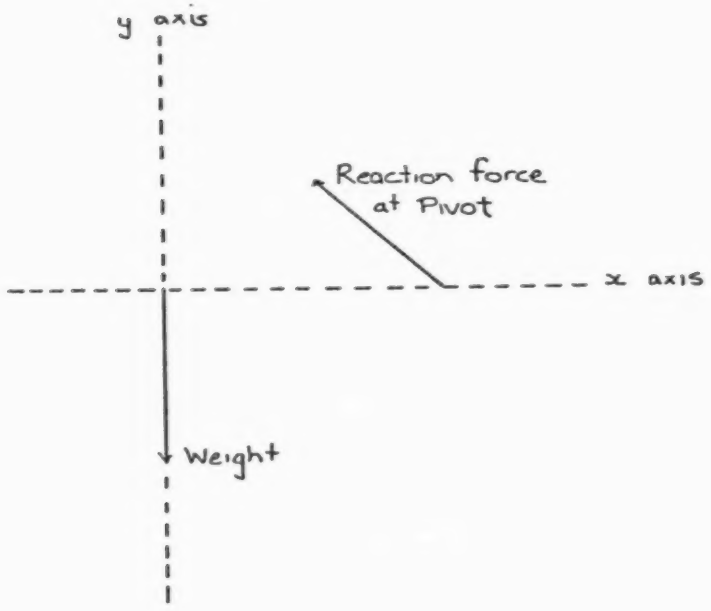


Figure 50

Drawbridge: Vectorgraph (ii)

STAGE 7 - ORTHOGRAPH

The orthograph stage requires vertical and horizontal component representation of forces in the vectorgraph on a graph reference frame.

Steps for Student Progress

1. Set up vertical and horizontal axes or equivalent orthogonal axes, depending on the nature of the problem.
2. Resolve the vectorgraph forces into horizontal and vertical components on the axes.
3. Draw these components in proper relation to each other on the graph axes.
4. Label the relevant dimensions, angles and component forces.

ORTHOGRAPH

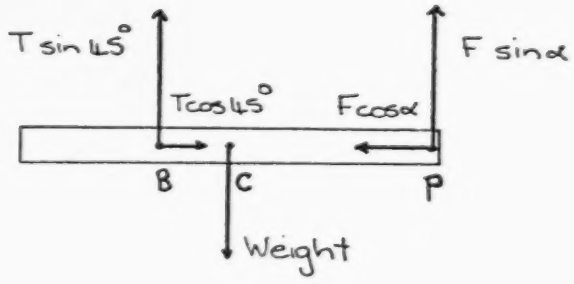


Figure 51

Drawbridge: Orthograph

5. Use the properties of triangles (we have an angle of 45^{0}; therefore, T sin 45° = T cos 45° in magnitude).
6. Apply the law of moments about the pivot point, to find T sin 45° (sum of the clockwise moments is equal and opposite to the sum of the anticlockwise moments).
7. Find F sin α by taking moments about B. Therefore,
   F sin α = T cos 45°.

STAGE 8 - SOLUTION STAGE

This stage requires the application of conditions for equilibrium. It takes into account the use of moments about relevantly selected points, which will reduce the number of unknowns in any equation to a minimum.

Steps for Student Progress

1. From the orthograph diagram, extract and write down equations of equilibrium.
2. Take moments about the pivot:
   (T sin 45°) BP = (weight) CP
   Since T is the only unknown, the tension, T can be evaluated.
3. Find the value of pivot force F and its direction angle, α.
4. To do this, two equations are necessary -
   Either:
   i) T sin 45° + F sin α = weight
   ii) T cos 45° = F cos α
   whence, F and α can be calcuated
   Or:
   i) (Weight) BC = (F sin α) BP
   ii) T cos 45° = F cos α
   whence F and α may be calculated.

STAGE 9 - VECTORGON (i) (Alternative)

This stage requires construction of the closed equilibrium polygon of forces. Properties of triangles may then be applied to solve required vectors.

Steps for Student Progress

1. The known vertical tension T_Y is laid off to a convenient scale.
2. Draw a line of action representing the given direction of tension T_X through the tip of the vertical vector T_Y and perpendicular to it.
3. Draw a line from the tip of the vector ray T_X to the tip of the vector ray T_Y.
4. This line represents the size and direction of resultant tension T_R.
5. The intersection of the three lines of action represent the vectors. It establishes the magnitude of the known component tensions, which are necessary to make the vector sum equal to zero.

VECTORGON (i)

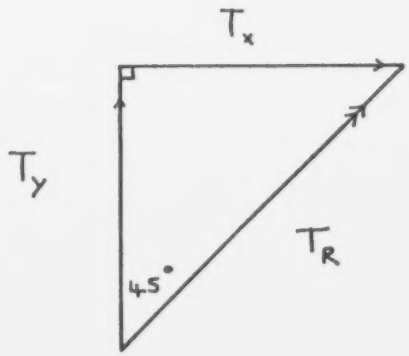


Figure 52

Drawbridge: Vectorgon (i)

6. The three tensions may now be scaled directly from the diagram.
7. Chain tension T_R may also be calculated using the properties of triangles - in this case, the pythagorean property of right triangles.
8. The chain tension is the first answer required in the problem statement.

STAGE 9 - VECTORGON (ii)

Steps for Student Progress

1. To find horizontal force F_H, we cannot use trigonometric properties of triangles in this case.
2. Use therefore, the zero resultant law of equilibrium :
   $\Sigma F_H = 0$.
3. Observe that the only forces operating along the horizontal axis are T_X and F_H.
4. Note that equilibrium is present.
5. Conclude therefore, that T_X is equal and opposite to F_H.
6. Draw to scale, F_H with magnitude T_X but in the opposite direction to vector T_X.
7. Draw to scale, F_V perpendicular to F_H.
8. Draw a line joining the extremities of the vector rays T_V and F_H.
9. This line of action represents the pivot reaction F_P of the drawbridge.

VECTORGON (ii)

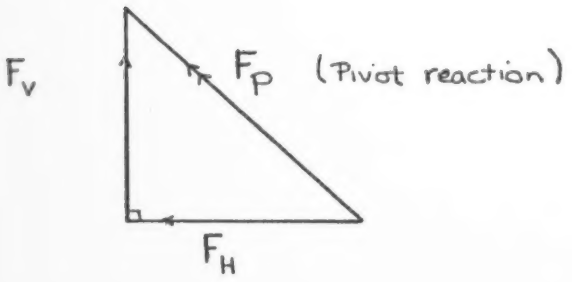


Figure 53

Drawbridge: Vectorgon (ii)

10. Pivot reaction F_P may now be scaled directly from the vectorgon.
11. F_P may also be calculated using the pythagorean property of right triangles.
12. The value of F_P is the second answer required by the problem statement.

The nine stage diagram analysis represents an approach for problem solving strategy, developed as a result of the findings in this study. The recommended strategy has originated from the exploratory research where no diagrammatic assistance at all was available. This formed the basic evidence of the need for a diagram analysis strategy. Such a strategy, it was determined, would need extremely detailed stage development. This was necessary to make obvious the linkage between the diagram stages. It was necessary, also, to provide guidance for students to make the correct transition between stages more independently as experience was gained. Hence, the list of steps for student progress.

As a general recommendation, the application of diagram analysis presents a logical opportunity for an improved physics problem solving strategy. As a specific teaching method, it has yet to be applied, tested and evaluated within the secondary classroom situation. Such evaluation research would appear to be the logical consequence of the findings of this study.

In final review then, the cognitive process of diagram analysis provides a recommendation for the redirection of emphasis in the teaching and learning strategy for physics problem solving. It attempts to approach more closely the proven concrete levels of student reasoning. The recommendations also confront and attempt to lessen the proven difficulties that student problem solvers have in the mental transition required between concrete operational and formal reasoning processes.

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APPENDIX A

MECHANICS VECTOR ANALYSIS TEST

Hayman Road
South Bentley
Western Australia 6102
Telephone (09) 350 7700
Telex AA 92983

Western Australian Institute of Technology

JD:PC

24 September 1981

File No.:
Reply to:
Your Ref.:

Dear Physics Student,

This booklet contains work on the T.A.E. Physics course of Western Australia. Each page contains a series of questions to help discover difficulties often met by physics students in solving problems in Mechanics.

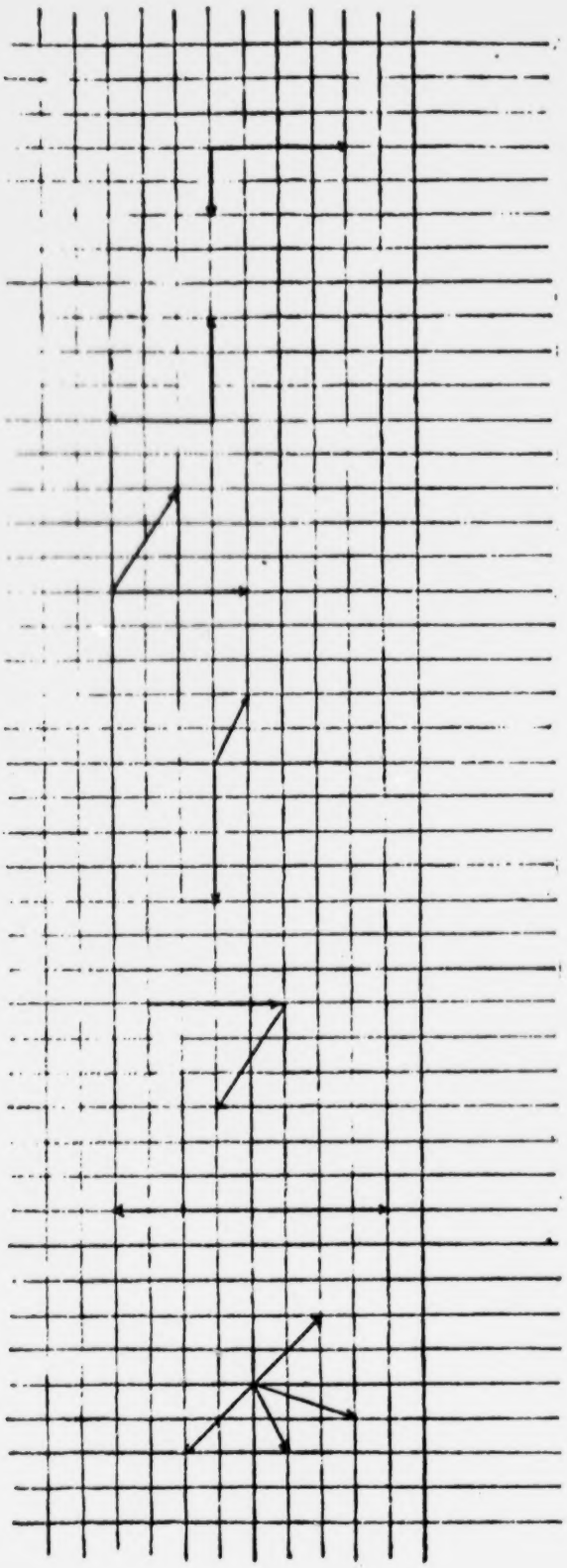
From this we hope to develop more helpful problem solving strategies for students at the secondary and tertiary level.

J. DEKKERS
J.A. DEACON
W.F. PRENDERGAST

1.

Each pair of forces below is drawn to scale.

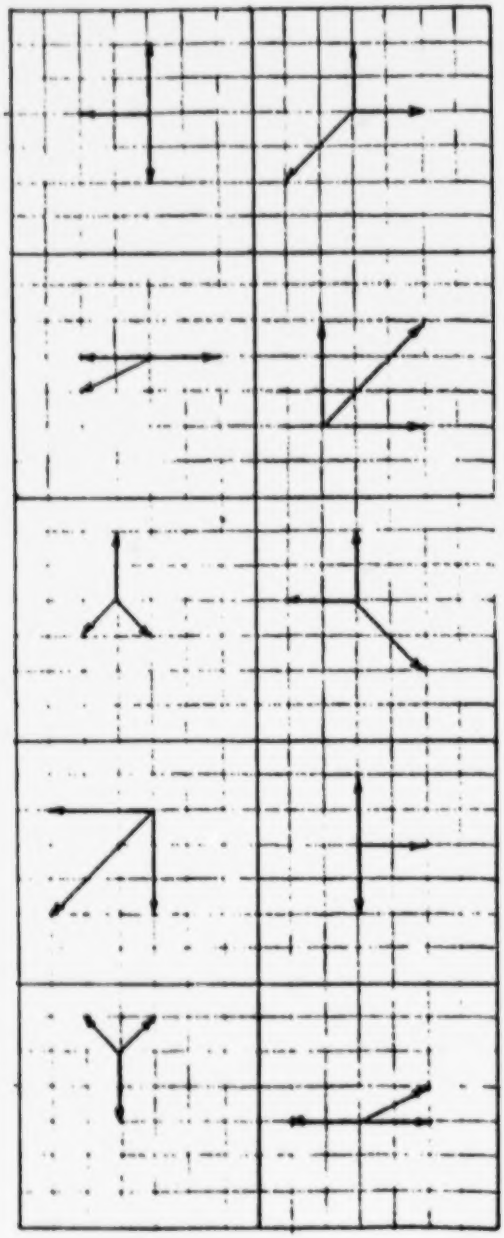
Sketch the resultant force in each case.



The diagrams each show 3 forces drawn to scale.

Tick (✓) the boxes where the forces <u>are</u> in equilibrium.

Cross (X) the boxes where the forces <u>are not</u> in equilibrium.

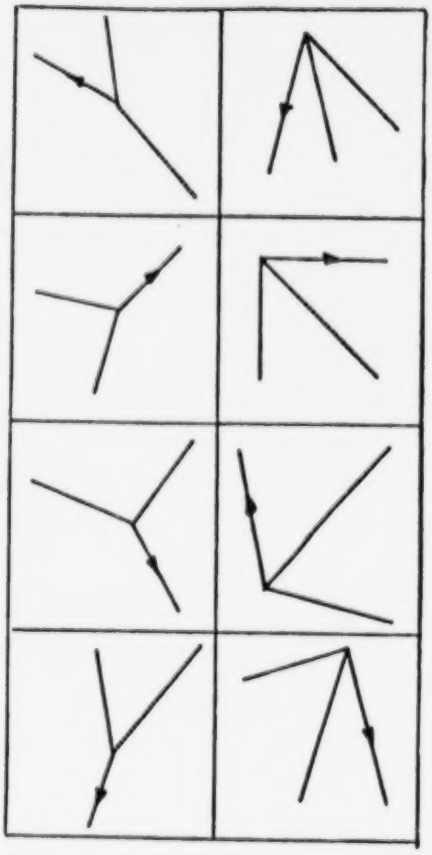


The diagrams below show sets of forces.

The force magnitudes are not drawn to scale.

Mark directional arrowheads on each set of forces in all diagrams, so that the direction chosen will create a static equilibrium.

When finished, <u>each</u> line should have <u>one</u> arrowhead.



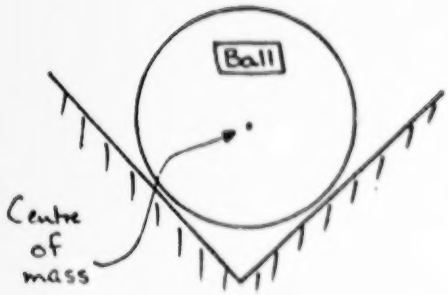
In providing your answer to each diagram opposite state or describe or define the physics principle(s) that you used.

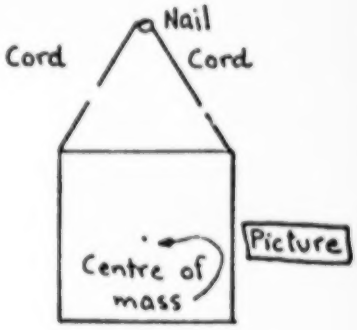
The word in the box names the body in static equilibrium.

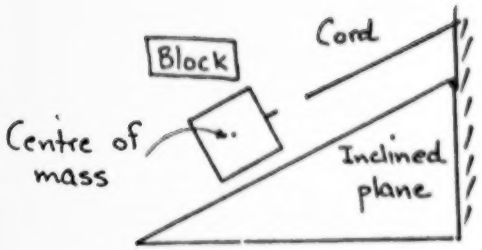
Draw the 3 forces which keep <u>this body</u> in equilibrium.

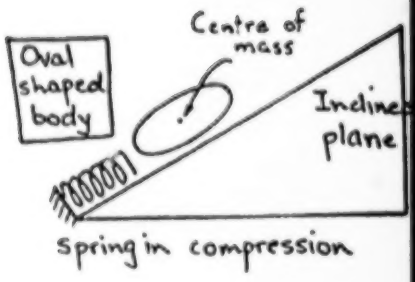
Show the direction and point of intersection of the lines of action of these forces.

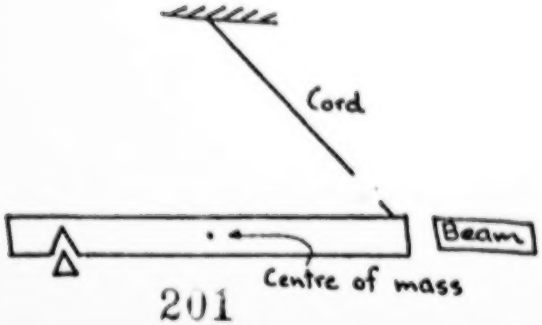
<u>NOTE</u>: Each drawing involves an exploded diagram wherein the positions of contact have been artifically separated for clarity.











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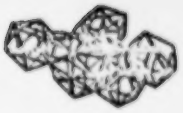


Western Australian Institute of Technology

JD:PC

24 September 1981

File No.:
Reply to:
Your Ref.:

TEST SOLUTIONS

Dear Physics Student,

This booklet contains work on the T.A.E. Physics course of Western Australia. Each page contains a series of questions to help discover difficulties often met by physics students in solving problems in Mechanics.

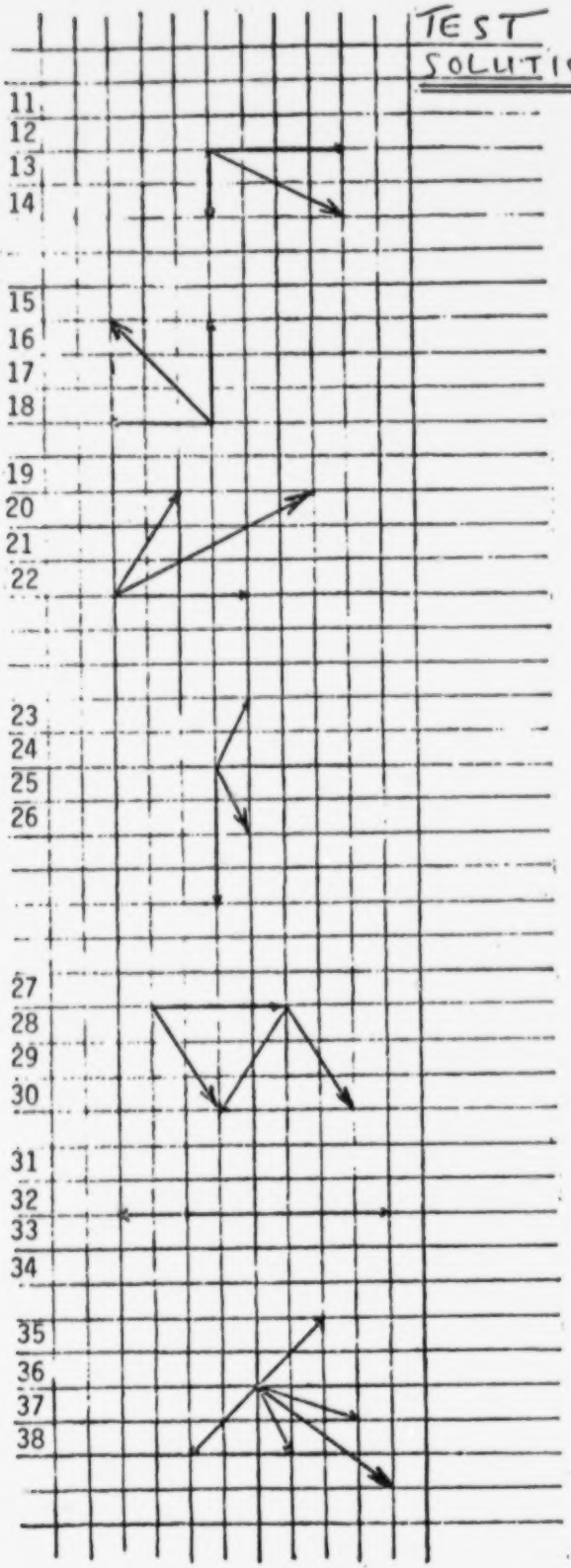
From this we hope to develop more helpful problem solving strategies for students at the secondary and tertiary level.

J. DEKKERS
J.A. DEACON
W.F. PRENDERGAST

Each pair of forces below is drawn to scale.

Sketch the resultant force in each case.

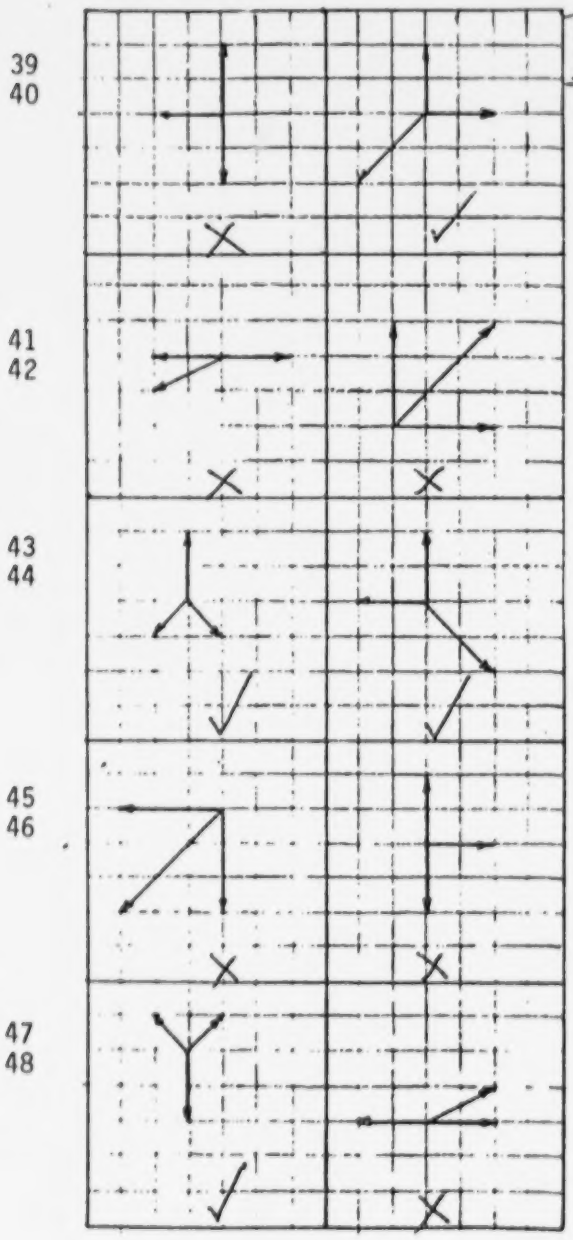
Point of Application
Line of Action
Vector Magnitude
Arrowhead Direction



The diagrams each show 3 forces drawn to scale.

Tick (✓) the boxes where the forces <u>are</u> in equilibrium.

Cross (X) the boxes where the forces <u>are not</u> in equilibrium.



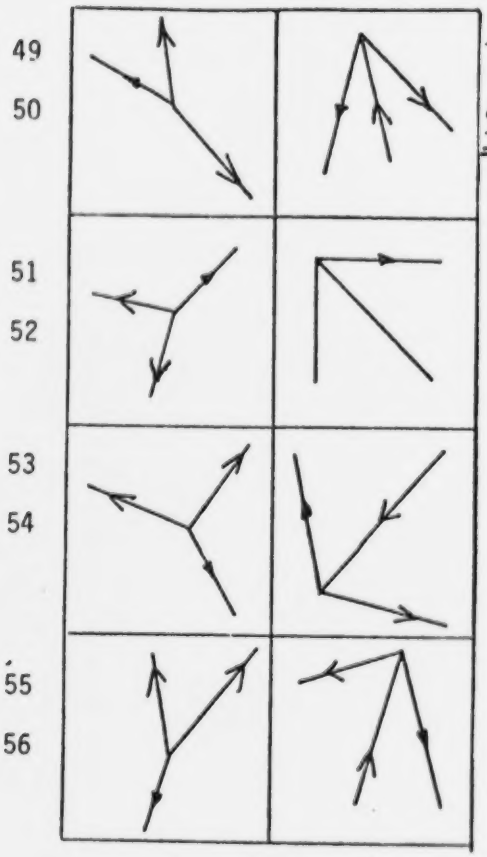
TEST SOLUTIONS

The diagrams below show sets of forces.

The force magnitudes are not drawn to scale.

Mark directional arrowheads on each set of forces in all diagrams, so that the direction chosen will create a static equilibrium.

When finished, each line should have one arrowhead.



TEST SOLUTIONS

In providing your answer to each diagram opposite state or describe or define the physics principle(s) that you used.

The word in the box names the body in static equilibrium.

Draw the 3 forces which keep this body in equilibrium.

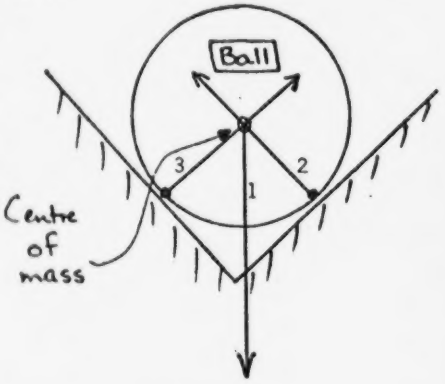
Show the direction and point of intersection of the lines of action of these forces.

NOTE: Each drawing involves an exploded diagram wherein the positions of contact have been artifically separated for clarity.

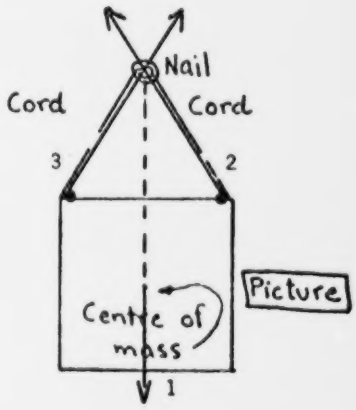
Proceed anticlockwise commencing with position of point of application in gravity vector

Point of application
Line of action
Arrowhead direction
Point of concurrence (triple inter-section)

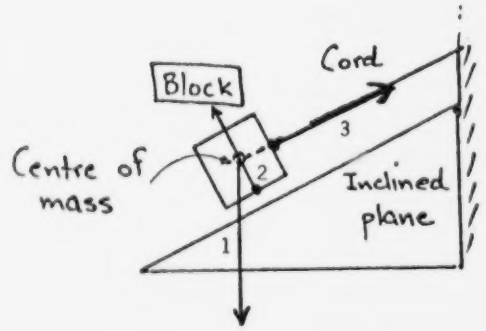
TEST SOLUTIONS



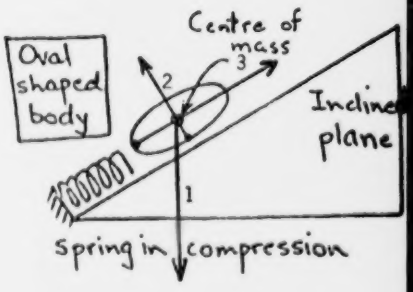
Problem 1



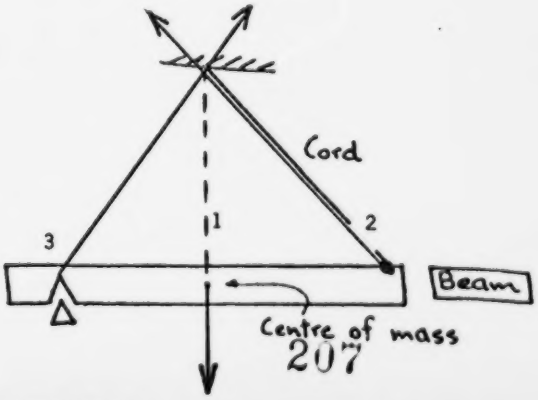
Problem 4



Problem 2



Problem 5



Problem 3

APPENDIX B

ANALYSIS OF VARIANCE

APPENDIX B

TABLE 26

ANALYSIS OF VARIANCE

MECHANICS VECTOR ANALYSIS TEST

Source of Variation	SS	DF	Mean Square	F	Probability
Between examinees	102·7	159	0·65		
Within examinees	1,560·3	9,280	0·17		
Between measures	547·6	58	9·44	85·96	0·00
Residual	1,012·76	9,222	0·11		
Nonadditivity	2·03	1	2·03	18·5	0·00
Balance	1,010·73	9,221	0·11		
Total	1,663·05	9,439	0·18		

Alpha (for examinees) = 0·83

Alpha (for items) = 0·84